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Review Article

# COMPARATIVE ANALYSIS OF NON LINEAR ESTIMATION SCHEMES USED FOR UNDERSEA SONAR APPLICATIONS

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#### ABSTRACT

The performance evaluation of various passive underwater target tracking algorithms such as pseudo-linear estimator, maximum likelihood estimator, modified gain bearings-only extended Kalman filter (MGBEKF), unscented Kalman filter, parameterized MGBEKF (PMGBEKF), and particle filter coupled with MGBEKF using bearings-only measurements is carried out with various scenarios in Monte Carlo simulation. The performance of PMGBEKF is found to be better than all estimates.

Keywords: Estimation, Target, Tracking, Sonar, Simulation, Ownship, Scenarios.

#### INTRODUCTION

Surveillance is the most important feature of maritime warfare and is undertaken by active as well as passive sensors. Active methods of surveillance require acoustic transmissions to be made by the surveillance platform and hence susceptible to interception by others. Hence, in certain situations, it becomes necessary to maintain silence on active mode. In the ocean environment, two-dimensional bearingsonly target motion analysis (TMA) is generally used. An ownship monitors noisy sonar bearings from a radiating target and finds out target motion parameters (TMP) - viz., range, course, bearing, and speed of the target. The basic assumptions are that the target moves at constant velocity most of the time. The ownship motion is unrestricted. The target and ownship are assumed to be in the same horizontal plane. The problem is inherently nonlinear as the measurement is nonlinear. The determination of the trajectory of a target solely from bearing measurements is called bearings-only tracking (BOT). The BOT area had been widely investigated and numerous solutions for this problem had been proposed [1]. In underwater, the ownship can be ship or submarine and the target will be submarine, ship or torpedo. Hence, there will be six types of ownship and target scenarios. It is observed pseudo-linear estimator (PLE), maximum likelihood estimator (MLE), modified polar coordinate extended Kalman filter (MPEKF), modified gain bearings-only extended Kalman filter (MGBEKF), unscented Kalman filter (UKF), parameterized MGBEKF (PMGBEKF), and particle Filter coupled with MGBEKF (PFMGBEKF) are successful contributions to this field. These days, very high processor based hardware is available. Hence, the algorithm optimization with respect to kilo lines of code, number of iterations, execution time, etc., is of minor importance. In practical applications, accuracy in the estimated solution and the number of samples required for convergence of the solution are of prime importance for the evaluation of the algorithms. In this paper, the required accuracy in the estimated solution is assumed. Hence, the purpose of this paper is performance evaluation of PLE, MLE, MPEKF, MGBEKF, UKF, PMGBEKF, and PFMGBEKF algorithms with respect to accurate convergence of the solution. The algorithms are evaluated with the scenarios shown in Table 1. In scenarios 1, 2 and 3, the ownship is assumed to be submarine and target is assumed to be submarine, ship and torpedo, respectively. Similarly in scenarios 4, 5 and 6, the ownship is assumed to be ship and target is assumed to be submarine, ship and torpedo, respectively. The target range and speeds are chosen as per the scenario. It means that for scenario 1, a submarine and submarine encounter, ownship speed is considered as 3.09 m/s and target speed as 4.12 m/s and initial range as 5 km. In all scenarios, the RMS error in bearing is assumed to be 0.33°. The algorithms are also evaluated to high bearing error (i.e. lower SNR) of the magnitude of 0.66° RMS, as worst condition. In underwater, sometimes outliers in the measurements are inevitable. Hence, it is assumed that 5% of the measurements are with 5 times of the error that is 1.65° (5\*0.33°) RMS. All these algorithms are evaluated against outliers also. The algorithms are realized through software, and the results in Monte Carlo simulation are presented.

A brief discussion of these algorithms is carried out in section 2. The results are presented and the performance evaluation of the algorithms against acceptance criteria is carried out in section 3. Finally, the paper is concluded in section 4.

## BRIEF DISCUSSION ON PASSIVE TARGET TRACKING ALGORITHMS

#### PLE

There is a development of PLE using an EKF [1] for passive target tracking using bearings-only measurements. Although this offers biased estimate at long ranges in certain scenarios, it has a practical advantage as it never diverges. The sophisticated estimators like MLE, require initial state estimates. Instead of choosing some arbitrary values, the PLE can be used to generate initial estimates for these estimators. Here, PLE is presented such that it does not require any initial estimate at all and at the same time offers sequential processing and flexibility to adopt the variance of each measurement.

The relevant equations of PLE in batch processing were presented [1]. The solution of the gradient equation was obtained by a Gauss-Newton iteration scheme. In Rao's study [2], PLE in batch processing was converted to sequential processing to suit real-time underwater applications such as passive target tracking by modifying the equations. All the elements of the covariance matrix were represented recursively in terms of the measurement equation. These were known as recursive sums and are maintained throughout the algorithm. This approach avoids computational complexity by computing only the incremental values for every new bearing measurement. These incremental values were used to update the recursive sums in the covariance matrix. Only a few recursive sums were updated on the arrival of a new bearing measurement. This method does not increase the computational burden even with an additional number of samples. Detailed mathematical modeling with simulation and results of PLE are available in Rao's study [2]. In this paper, PLE is used to compare its performance with those other standard estimators for passive target tracking application.

Table 1: Scenarios chosen for evaluation of algorithms

Encounter	Scenario	Initial range (m)	Initial bearing (deg)	Target speed (m/s)	Ownship speed (m/s)	Target course (deg)
Submarine to Submarine	1	5000	0	4.12	3.09	135
Submarine to Ship	2	20000	0	12.36	3.09	135
Submarine to Torpedo	3	18000	0	20.6	3.09	135
Ship to Ship	4	20000	0	12.36	12.36	135
Ship to Submarine	5	5000	0	3.09	12.36	135
Ship to Torpedo	6	20000	0	20.6	12.36	135

#### MLE

MLE using batch processing for passive target tracking was developed [1]. In Rao's study [3], MLE in batch processing was converted into sequential processing. The procedure used for conversion was similar to that of PLE.

MLE requires an initial estimate to estimate TMP. Instead of assuming some arbitrary values for initialization, PLE's outputs are utilized as initial estimates for MLE. As PLE generates bias in the estimates, its use is restricted to generate a reasonably accurate estimate for initialization of MLE. In this paper, MLE is used to compare its performance with those other standard estimators for passive target tracking application.

#### **MPEKF**

In MPEKF [4], the pertinent equations of state and measurement were formulated in modified polar coordinates, while the algorithm itself was configured as an EKF. This coordinate system is shown to be well suited for bearings-only TMA because it automatically decouples observable and unobservable components of the estimated state vector. Such decoupling prevents covariance matrix ill-conditioning, which is the primary cause of filter instability. Further investigation also confirmed that the resulting state estimates were asymptotically unbiased, as required. It calculates the smoothed bearing and bearing rate, which are very useful for target tracking application. The modified polar state vector was comprised the following four componentsbearing, bearing rate, range rate divided by range and the reciprocal of range. In theory, the first three components can be determined from single-sensor bearing data without an ownship manoeuvre; the fourth component, however, should remain unobservable until this manoeuvre requirement is satisfied. These theoretical properties are implicitly preserved in the modified polar filter formulation. In essence, the state estimates are constrained to behave as predicted by theory, even in the presence of errors in measurements. Under similar conditions, standard Cartesian filters often experience covariance matrix ill-conditioning which precipitates false observability. In this paper, MPEKF is used to compare its performance with those other standard estimators for passive target tracking application.

### Modified gain extended Kalman filter (MGEKF)

The divergence in EKF [5] was eliminated by modifying the gain function, and this algorithm is named as MGEKF [6]. This algorithm is another successful contribution to this field. The essential idea behind MGEKF is that the nonlinearities be "modifiable." This algorithm has some similarities with the pseudo measurement function but not the same. In pseudo measurement filter, the gain is a function of past and present measurements. It is to be noted that MGEKF is based on EKF algorithm, and the gain of the MGEKF is a function of only past measurements. By eliminating the direct correlation of the gain and measurement noise process in the estimates of MGEKF, the bias in the estimation is avoided. A simplified version of the modified gain functionis available in Galkowski and Islam's study [7]. This version is useful for air applications, where elevation and bearing measurements are available. In underwater, bearings-only measurements are available. MGEKF is further modified for underwater applications, and the algorithm is named as MGBEKF [8,9]. In this paper, its performance is analyzed for ocean environment in which the vehicles move at low speeds and the measurements are corrupted with high noise. In this paper, MGBEKF is used to compare its performance with those other standard estimators for passive target tracking application.

#### UKF

The traditional Kalman filter is optimal when the model is linear. Unfortunately, many of the state estimation problems like tracking of the target using bearings-only information are nonlinear, thereby limiting the practical usefulness of the Kalman filter and EKF. Hence, the feasibility of a novel transformation, known as unscented transformation, which is designed to propagate information in the form of mean vector and covariance matrix through a nonlinear process, is explored for underwater applications. The unscented transformation is coupled with certain parts of the classical Kalman filter. It is easier to implement and use the same order of calculations [10]. UKF can be treated as an alternative to MGBEKF. But still, the basic constraint is that the probability density function of noise in the measurements is to be Gaussian for optimum results. UKF can take up nonlinearity but not non-Gaussian noise in the measurements. In a study by Rao and Rao [11,12], detailed mathematical modeling with application to BOT is available. In this paper, UKF is used to compare its performance with those other standard estimators for passive target tracking application.

#### Particle filter

Particle filter [13-16] is the new generation advanced filter, which is useful for nonlinear and non-Gaussian applications. Particle filter uses a set of weighted state samples, called particles, to approximate the posterior probability distribution in a Bayesian setup. At any point of time, the set of particles can be used to approximate the PDF of the state. As the number of particles increase to infinity, the approximation approaches the true PDF. They provide nearly optimal state estimates in the case of nonlinear and non-Gaussian systems, unlike Kalman filter based approaches. Because particle filter does not approximate nonlinearities or non-Gaussian noise in the system and use a large number of particles, they tend to be computationally complex. However, with the currently available advanced microprocessors, the computation can be easily managed. The basic idea of the particle filter is as follows. It was invented to numerically implement the Bayesian estimator. The main idea is intuitive and straight forward. At the beginning of the estimation problem, N state vectors are randomly generated based on the initial PDF  $p(X_c(0))$  (which is assumed to be known). These state vectors are called particles and are denoted as  $X_c(k,k)$  k=1, 2 ..., N. At each time step, the particles are propagated to the next time step using the process equation.

$$X_{S}(k+1,k)=f(X_{S}(k,k), \omega(k+1)), k=1, 2,..., N$$
 (1)

Where plant noise,  $\omega$  is randomly generated on the basis of its known PDF. After receiving the measurement at time k, the conditional relative likelihood of each particle,  $X_s(k+1,k)$  is computed. That is, the PDF  $p(Z(k),X_s(k+1,k))$  is evaluated. This can be done if the nonlinear measurement equation and the PDF of the measurement noise are known. For example, if an m-dimensional measurement equation is given as  $Z(k)=h(X_s(k))+\gamma_g(k)$ , and then a relative likelihood q(k) can be computed as follows [15].

$$q(k) = P\left[Z(k) = z^*, X_S(k) = X_S(k+1,k)\right]$$

$$= P\left[\gamma(k) = z^* - h(X_S(k+1,k))\right]$$

$$\sim \frac{1}{(2\pi)^{m/2} (\sigma_B^2)^{m/2}} \times \exp\left[\frac{-\left[z^* - h(X_S(k+1,k))\right]^T \left[z^* - h(X_S(k+1,k))\right]}{2\sigma_B^2}\right]$$
(2)

The  $\sim$  symbol in the above equation means that the probability is not really given by the expression on the right side, but the probability is directly proportional to the right side. Hence, if this equation is used for all the particles,  $X_s(k+1,k)$  (k=1,2,...,N), then the relative likelihoods that the state is equal to each particle will be correct. Now the relative likelihoods obtained are normalized as follows:

$$q(k) = \frac{q(k)}{\sum_{i=1}^{N} q(i)}$$
(3)

Then, the particles using the computed likelihoods are resampled. This means a new set of particles are randomly generated on the basis of the relative likelihoods q(k).

#### 2.6.1 PARTICLE FILTER COMBINED WITH OTHER FILTERS

One approach that has been proposed for improving particle filtering is to combine it with another filter such as the EKF, UKF, or MGBEKF [15]. In this approach, each particle is updated at the measurement time using the EKF, UKF, or MGBEKF and then resampling (if required) is performed using the measurement. This is like running a bank of Kalman filters (one for each particle) initialized with randomly chosen state vectors and then adding a resampling step (if required) after each measurement. After  $X_s(k+1,k)$  is obtained, it can be refined using the EKF, UKF, or MGBEKF measurement-update equations. In this thesis particle filter is combined with the MGBEKF and the algorithm is named as PFMGBEKF.  $X_s(k+1,k)$  is updated to  $X_s(k+1,k+1)$  according to the following MGBEKF equations [15].

$$\begin{split} P(k+1,k)_{i} &= \varphi(k+1,k)_{i} \, P(k,k)_{i} \, \varphi^{T} \left(k+1,k\right)_{i} + \Gamma Q(k+1) \Gamma^{T} \\ G(k+1)_{i} &= P(k+1,k)_{i} \, H^{T} \left(k+1\right)_{i} \\ & \left[ \sigma_{B}^{2} + H(k+1)_{i} \, P(k+1,k)_{i} \, H^{T} \left(k+1\right)_{i} \right]^{-1} \\ X_{S}\left(k+1,k+1\right)_{i} &= X_{S}\left(k+1,k\right)_{i} + G(k+1)_{i} \\ & \left[ B_{m}(k+1) - h(k+1,X_{S}\left(k+1,k\right)_{i}\right) \right] \\ P(k+1,k+1)_{i} &= \left[ I - G(k+1)_{i} \, g \left(B_{m}(k+1),X_{S}\left(k+1,k\right)_{i}\right) \right] P(k+1,k)_{i} \\ \times \left[ I - G(k+1)_{i} \, g \left(B_{m}(k+1),X_{S}\left(k+1,k\right)_{i}\right) \right]^{T} + \sigma_{B}^{2} G(k+1)_{i} \, G^{T}\left(k+1\right)_{i} \end{split}$$

Where G(k+1) is Kalman gain, P(k+1,k) is a priori estimation error covariance for the  $i^{th}$  particle and g(.) is modified gain function. g(.) is given by

$$g = \begin{bmatrix} 0 & 0 & \cos B_m / (\hat{R}_x \sin B_m + \hat{R}_y \cos B_m) \\ -\sin B_m / (\hat{R}_x \sin B_m + \hat{R}_y \cos B_m) \end{bmatrix}$$
(5)

Since true bearing is not available in practice, it is replaced by the measured bearing to compute the function g(.).

#### Resampling

In every update of PFMGBEKF, it is monitored to decide whether resampling of particles in respect of target state vector and its covariance matrix is required or not. Resampling is required when the effective sample size,  $N_{eff} < N/3$  [15].

Where,

$$N_{eff} = \frac{1}{\sum_{i=1}^{N} q_i^2} \tag{6}$$

Whenever resampling is required, the following procedure based on weight of particles is adopted. In this method, weights are sorted in descending order. The corresponding original indexes before sorting are remembered. Then, replication of particles (both the state and covariance matrices) is carried out in proportion to the weight of particles starting with the particle with maximum weightage. This procedure is repeated for the particle with the next maximum weightage. This process is continued till all the particle positions are filled up. This method is close to the method suggested in Simon's study [15].

#### **PMGBEKE**

The work presented in Ristick *et al.*'s study [14] is found interesting. Ristick *et al.*'s study [14] divided the range interval of interest into a number of sub-intervals following geometric progression and each sub-interval was dealt with an independent Kalman filter. They suggested that this method can be extended to course and speed parameterization, if prior knowledge of target course and speed respectively are vague. Parameterization in initialization reduces the dependence of convergence of the solution on initialization. In underwater scenario, prior knowledge of target range, course and speed is vague.

In this situation, obtaining fast convergence has an important role and this is achieved using parameterization. Inclusion of range, course and speed parameterization is proposed for MGBEKF to track a torpedo using bearings-only measurements. This algorithm is named as PMGBEKF.

Let the range, course and speed intervals of interest be (maximum-range, minimum-range), (maximum-course, minimum-course) and (maximum-speed, minimum-speed), respectively. The initial weights of each MGBEKF are set to 1/N, subsequently, the weight of filter i at time k is given by

$$\varsigma^{i}(k) = \frac{p(B(k),i)\varsigma^{i}(k-1)}{\sum_{j=1}^{N} p(B(k),j)\varsigma^{j}(k-1)}$$
(7)

Where, p(B(k),i) is the likelihood of measurement B(k). Assuming Gaussian statistics, the likelihood p(B(k),i) can be computed as:

$$p(B(k),i) = \frac{1}{\sqrt{2\pi\alpha^{i}_{inv}^{2}}} \exp\left[-\frac{1}{2} \left(\frac{B(k) - \hat{B}^{i}(k,k-1)}{\sigma^{i}_{inv}}\right)^{2}\right]$$
(8)

where  $\hat{B}^i(k,k-1)$  is the predicted angle at k for filter i and  $\sigma^i_{inv}^2$  is the innovation variance for filter i given by

$$\sigma_{inv}^{i}^{2} = \hat{H}^{i}(k)P^{i}(k,k-1)\hat{H}^{iT}(k) + \sigma_{B}^{2}$$
(9)

where  $\hat{H}^i(k)$  is the Jacobian of nonlinear measurement function and  $P^i(k,k-1)$  is the predicted covariance for filter i. Let the state estimate

of filter i be  $\hat{X}^i(k,k)$  and its associated covariance be  $P^i(k,k)$ , then the combined estimate of PMGBEKF is computed using the Gaussian mixture formulas [14] as follows.

$$\hat{X}(k,k) = \sum_{i=1}^{N} \varsigma^{i}(k) \hat{X}^{i}(k,k)$$

$$P(k,k) = \sum_{i=1}^{N} \varsigma^{i}(k) \left[ P^{i}(k,k) + \left( \hat{X}^{i}(k,k) - \hat{X}(k,k) \right) \left( \hat{X}^{i}(k,k) - \hat{X}(k,k) \right)^{T} \right]$$
(11)

#### SIMULATION AND RESULTS

Simulator is developed to create target, ownship and measurements. It is assumed that the ownship is at the origin and bearing is considered with respect to Y-axis,  $0-360^{\circ}$  and clockwise positive. Target and ownship movements are updated at every second. All one second samples are corrupted by additive zero mean Gaussian noise. It is assumed that the bearing measurements are available continuously at every second. The ownship is assumed to be carrying out S-manoeuver with a turning rate of  $1^{\circ}$ /s. The ownship moves initially at a course of  $90^{\circ}$  for a period of 2 min, and then, it changes to course  $270^{\circ}$ . At  $9^{\text{th}}$ ,  $16^{\text{th}}$ , and  $23^{\text{rd}}$  min, the ownship changes its course from  $270-90^{\circ}$ ,  $90-270^{\circ}$  and  $270-90^{\circ}$ , respectively, as shown in Fig. 1. The experiment is conducted for 1000 s.

#### Initialization of state vector and its covariance matrix

In PLE, algorithm initialization of target state vector is not required. In MLE algorithm PLE's outputs obtained after ownship first manoeuvre is used for initialization of MLE. Let the sonar range of the day be 20 km that means sonar can detect the ship at the range of maximum 20 km on that particular day. Using this information in MGBEKF, UKF

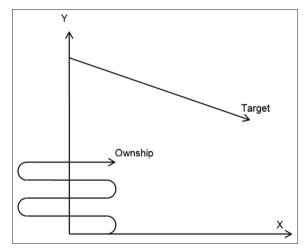


Fig. 1: Ownship in S-manoeuvre

and PFMGBEKF, target state vector position components are initialized with 20 km. As the speed of the target is not available, the velocity components of target are each assumed as 10 m/s. (It is known that the submarine target moves at around 3 m/s and torpedo moves at around 17 m/s. As the same algorithm is to be used to track ship, submarine and torpedoes, average speed of the underwater vehicles is considered). In PF, it is observed that around 10,000 particles are necessary to obtain good results. When PF is combined with MGBEKF, 1000 particles are sufficient to get the required accuracy in the solution. (It is also seen that by increasing the particles to 10,000 there is no improvement in accuracy of the solution). In PMGBEKF, the range, course and speed sets contain 3-20 km, 0-359° and 3-20 m/s, respectively. The elements of range, course and speed sets follow geometric progression. In MPEKF algorithm, the target state vector in modified polar coordinates is initialized using the above said position and velocity components.

It is assumed that initialized target state vector follows uniform density function. Accordingly, the covariance matrix of initial target state vector components is derived for MGBEKF, UKF, and PFMGBEKF. In the case of MPEKF, it is assumed that the variance of course and speed are 0.5° and 0.1 m/s, respectively, and the covariance matrix is derived as given in Aidala and Hammel's study [4].

#### Performance evaluation of the algorithms

It is assumed that the TMP are said to be converged when the error in the range, course and speed estimates are less than or equal to 10% of the actual range, 5° of the actual course and 20% of the actual speed, respectively. As mentioned earlier, in PFMGBEKF-1000 KF's are used. In PMGBEKF range, course and speed sets with 5 elements (in geometric progression) each are used and so 125 KF's work in parallel. Although it takes more execution time when compared to with that of PLE, MLE, MGBEKF, UKF and less execution time with that of PFMGBEKF, execution time is not considered to select as right algorithm for passive target tracking as mentioned earlier. The convergence time to obtain the range, course and speed estimates together with the required accuracies using each algorithm in each scenario is shown in Table 2. From the results obtained, it is evident PMGBEKF estimates the solution faster when compared to that of other estimators. For robustness, PMGBEKF is tested for the following cases namely-lower SNR and outliers. For the purpose of presentation of the results, the bearing error is increased from 0.33° to 0.66° RMS in scenario 1 and the results obtained are shown in Table 2. It is assumed that 5% outliers in underwater do exist and so 5% of the measurements are randomly chosen with 1.65° (5\*0.33) RMS error. Again Scenario 1 is chosen with outliers and the results obtained are shown in Table 2.

### **Detailed analysis**

Scenario 1 is chosen for the presentation of the results in detail. The convergence time for range, course and speed estimates with  $0.35^\circ$  RMS,  $0.66^\circ$  RMS and 5% outliers with  $1.65^\circ$  RMS error in bearing measurements is shown in Table 3. The estimates of range, course, and speed when the error is  $0.33^\circ$  RMS in bearing measurements are plotted with respect to time in Figs. 2a and b, 3a and b, 4a and b, respectively.

Table 2: Convergence time of various algorithms in seconds

Scenario	RMS error in bearing, deg	PLE	MLE	MPEKF	MGBEKF	UKF	PFMGBEKF	PMGBEKF
Scenario	KM3 error in bearing, deg	ILL	MLL	MII EKI	MUDLINI	UKI	TIMUDEKI	I MUDLIKI
1	0.33	477	459	465	461	462	408	362
	0.66	604	470	494	512	510	458	430
	5% outliers with 1.65	574	483	478	474	477	429	385
2	0.33	725	718	580	582	585	519	385
3	0.33	610	601	305	301	300	248	280
4	0.33	548	520	515	520	525	390	380
5	0.33	358	348	390	400	405	412	300
6	0.33	431	417	420	411	415	450	360

PLE: Pseudo linear estimator, MLE: Maximum likelihood estimator, MPEKF: Modified polar coordinate extended Kalman filter, MGBEKF: Modified polar coordinate extended Kalman filter, UKF: Unscented Kalman filter, PFMGBEKF: Particle filter coupled with modified gain bearings-only extended Kalman filter, PMGBEKF: Parameterized modified gain bearings-only extended Kalman filter

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Table 3: Convergence time in seconds for	range, course and speed estimates with scenario

RMS error in bearing, deg	Target parameters	PLE	MLE	MPEKF	MGBEKF	UKF	PFMGBEKF	PMGBEKF
0.33	Range	242	238	465	309	311	300	245
	Course	477	459	431	461	462	408	362
	Speed	344	315	428	411	409	400	301
0.66	Range	272	279	494	399	405	401	256
	Course	604	470	452	510	512	458	430
	Speed	584	323	441	430	427	410	315
5% outliers with 1.65	Range	263	266	478	319	325	360	246
	Course	574	483	433	474	477	429	385
	Speed	385	326	431	418	414	403	306

PLE: Pseudo linear estimator, MLE: Maximum likelihood estimator, MPEKF: Modified polar coordinate extended Kalman filter, MGBEKF: Modified polar coordinate extended Kalman filter, UKF: Unscented Kalman filter, PFMGBEKF: Particle filter coupled with modified gain bearings-only extended Kalman filter, PMGBEKF: Parameterized modified gain bearings-only extended Kalman filter

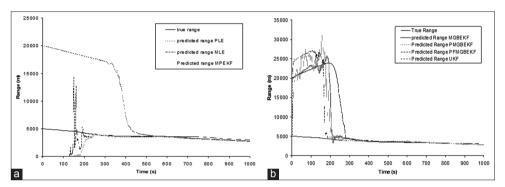


Fig. 2: (a) Estimated ranges of pseudo linear estimator, maximum likelihood estimator and modified polar coordinate extended Kalman filter algorithms. (b) Estimated ranges of modified polar coordinate extended Kalman filter, unscented Kalman filter, particle filter coupled with modified gain bearings-only extended Kalman filter and parameterized modified gain bearings-only extended Kalman filter algorithms

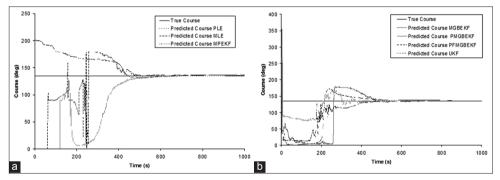


Fig. 3: (a) Estimated courses of pseudo linear estimator, maximum likelihood estimator and modified polar coordinate extended Kalman filter algorithms. (b) Estimated courses of modified polar coordinate extended Kalman filter, unscented Kalman filter, particle filter coupled with modified gain bearings-only extended Kalman filter and parameterized modified gain bearings-only extended Kalman filter algorithms

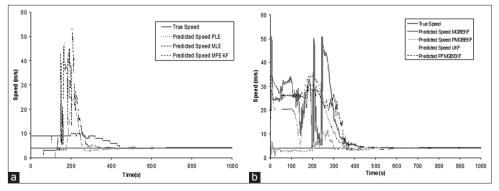


Fig. 4: (a) Estimated speeds of pseudo linear estimator, maximum likelihood estimator and modified polar coordinate extended Kalman filter algorithms. (b) Estimated courses of modified polar coordinate extended Kalman filter, unscented Kalman filter, particle filter coupled with modified gain bearings-only extended Kalman filter and parameterized modified gain bearings-only extended Kalman filter algorithms

From Tables 2 and 3, it is clear that MLE is better than PLE as expected. In fact, after observability of the process, the PLE outputs are used to initialize MLE target state vector. The performance of MGBEKF and UKF is almost same and it is observed that MGBEKF generates solution faster with few samples when compared to that of UKF. A similar statement is also reported in Rao's study [17]. The performance of MPEKF is better when compared to that of MLE. It is difficult to say which is better when compared with MGBEKF/UKF. The performance of PFMGBEKF is in between to that of UKF and MGBEKF. Undoubtedly PMGBEKF generates the solution faster. In PMGBEKF solution converges at around 330±50 s for all types of scenarios because of parameterization in target state vector.

#### SUMMARY AND CONCLUSION

In underwater, the ownship can be ship or submarine and the target will be submarine, ship or torpedo. Hence, there will be six types of ownship and target scenarios. In this chapter, six scenarios as shown in Table 1 are chosen covering the above said types. Various passive target tracking algorithms as shown in Table 2 are considered for the comparative study of performance evaluation of algorithms with respect to convergence of the solution. For robustness, the algorithms are tested against at low SNR and with outliers. Simulation is carried out and the results are presented in Table 2. It is observed that PMGBEKF generates the solution faster when compared to that of other estimators.

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