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Review Article

IMPROVED NONLINEAR SIGNAL ESTIMATION TECHNIQUE FOR UNDERSEA SONAR-BASED NAVAL APPLICATIONS

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ABSTRACT

The aim of this work is to develop passive target tracking algorithm, suitable for implementation in target motion analysis for underwater applications. The vehicle is assumed to be standstill in underwater watching for any target ship using bearings-only measurements. Using these measurements, the algorithm calculates the course of the target, which is further used to find out target range and speed. Provision is given to generate range and course if the speed of the target is known by some other means. Pseudo linear estimator (PLE) is developed to reduce the noise in the measurements and to find out target motion parameters. Although PLE offers a biased estimate in certain scenarios, it has an advantage as it hardly diverges. It offers the features of Kalman filter, viz., sequential processing, flexibility to adopt the variance of each measurement, etc. The Monte Carlo simulation results are presented for a typical scenario, and it is shown that this algorithm is useful for naval underwater applications.

Keywords: Target motion analysis, Doppler, Estimation, Sonar, Active signal transmission, Maritime, Line of sight measurements.

INTRODUCTION

In the ocean environment, two-dimensional bearings-only target motion analysis (TMA) is generally used. The underwater vehicle monitors noisy sonar bearings from a radiating target in passive listening mode and processes these measurements to find out target motion parameters (TMPs), *viz*., range, course, bearing, and speed of the target. As range measurement is not available and the bearing measurement is not linearly related to the target states, the whole process becomes nonlinear. Added to this, since bearing measurements are extracted from single passive sonar, the process remains unobservable until observer executes a proper maneuver.

Many researchers tried to eliminate this requirement of ownship maneuver to obtain the TMPs, using Doppler shifted tonal frequencies [1-3], elevation measurements [4], and one active range measurement along with bearing measurements [5]. I would like to introduce a new method, motivated by the paper written by Chan and Rea [5]. In this method, ownship maneuver is not required.

Section 2 describes the mathematical modeling of the pseudo linear formulation with sequential implementation. Simulation and results are presented in Section 3, and finally, the paper is concluded in Section 4.

MATHEMATICAL MODELING

It is desired to track the target using noise-corrupted bearing measurements. The target state equation is given by Nardone *et al*. [8].

$$X_s(k+1)=\phi(k+1,k)\,X_s(k)+b(k+1) \tag{1}$$

Where, X_s (k) is a state vector with target velocity and position components and is given by

$$X_s(k)=\begin{bmatrix}\dot{x}(k) & \dot{y}(k) & R_x(k) & R_y(k)\end{bmatrix}^T \tag{2}$$

$\phi(k+1|k)$ is a transient matrix and t is the sampling time between two successive measurements. The transient matrix is given by,

$$\varphi\,(k+1|k)=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ t & 0 & 1 & 0\\ 0 & t & 0 & 1\end{bmatrix} \tag{3}$$

The deterministic vector, b(k+1) is given by,

$$b(k+1)=\begin{bmatrix}0 & 0 & -\{x_0(k+1)-x_0(k)\} & -\{y_0(k+1)-y_0(k)\}\end{bmatrix} \tag{4}$$

The bearing measurement is given by,

$$B(k)=\tan^{-1}(R_x(k)/R_y(k)) \tag{5}$$

Where, $R_x(k)$ and $R_y(k)$ are the relative range components at instant k. Here, bearing $B_m(k)$ is considered with respect to true north and is given by,

$$B_m(k)=B(k)+\eta(k) \tag{6}$$

Where, η(k) is error in the measurement and is assumed to be zero mean Gaussian with variance σ^2. I would like to find out TMP in two modes. In the first mode, the vehicle is assumed to be standstill and the second mode it is assumed to be moving. Let us find out TMP in the first mode. In this mode, b(k+1) in equation (1) becomes zero. Substituting equation (5) in (6), we obtain,

$$R_x\,(k)\cos B_m\,(k)-R_y\,(k)\sin B_m\,(k)+\xi(k)=0 \tag{7}$$

Where, ξ(k) is given by,

$$\xi(k)=\eta(k)R(k) \tag{8}$$

Where, R(k) is the range between observer and target at time index k. The equation (5) can be written as,

$$H(k)X_s(k)+\xi(k)=0 \tag{9}$$

Where, H(k) is given by,

$$H(k)=\begin{bmatrix}0 & 0 & \cos B_m(k) & -\sin B_m(k)\end{bmatrix} \tag{10}$$

Let us assume, we are interested in finding out the TMPs through the initial state vector, then equation (9) can be modified as,

$$H(k)\varphi(k,0)X_s(0)+\xi(k)=0 \tag{11}$$

$$\text{Where, } \varphi(k,0)=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ kt & 0 & 1 & 0\\ 0 & kt & 0 & 1\end{bmatrix} \tag{12}$$

Considering k measurements, using equation (11), we can write that

$$\begin{bmatrix} t.\cos B_m(1) & -t.\sin B_m(1) & \cos B_m(1) & -\sin B_m(1)\\ 2t.\cos B_m(2) & -2t.\sin B_m(2) & \cos B_m(2) & -\sin B_m(2)\\ . & . & . & .\\ . & . & . & .\\ kt.\cos B_m(k) & -kt.\sin B_m(k) & \cos B_m(k) & -\sin B_m(k)\end{bmatrix} \tag{13}$$

$$\begin{bmatrix}\dot{x}_t\\ \dot{y}_t\\ R_x(0)\\ R_y(0)\end{bmatrix}+\begin{bmatrix}\xi(1)\\ \xi(2)\\ .\\ .\\ \xi(k)\end{bmatrix}=0$$

For the purpose of finding out target course, equation (13) is rewritten as,

$$\begin{bmatrix} t.\cos B_m(1) & -t.\sin B_m(1)\\ 2t.\cos B_m(2) & -2t.\sin B_m(2)\\ . & .\\ . & .\\ kt.\cos B_m(k) & -kt.\sin B_m(k)\end{bmatrix}\begin{bmatrix}\dot{x}_t\\ \dot{y}_t\end{bmatrix}+ \tag{14}$$

$$\begin{bmatrix}\cos B_m(1) & -\sin B_m(1)\\ \cos B_m(2) & -\sin B_m(2)\\ . & .\\ . & .\\ \cos B_m(k) & -\sin B_m(k)\end{bmatrix}\begin{bmatrix}R_x(0)\\ R_y(0)\end{bmatrix}+N(k)=0$$

Where, N(k) is given by,

$$N(k)=\begin{bmatrix}\xi(1)\\ \xi(2)\\ .\\ .\\ \xi(k)\end{bmatrix}$$

Equation (14) can be written as,

$$A'X_v+DX_p(0)+N(k)=0 \tag{15}$$

Where,

$$A'=\begin{bmatrix} t.\cos B_m(1) & -t.\sin B_m(1)\\ 2t.\cos B_m(2) & -2t.\sin B_m(2)\\ . & .\\ . & .\\ kt.\cos B_m(k) & -kt.\sin B_m(k)\end{bmatrix} \tag{16}$$

$$D=\begin{bmatrix}\cos B_m(1) & -\sin B_m(1)\\ \cos B_m(2) & -\sin B_m(2)\\ . & .\\ . & .\\ \cos B_m(k) & -\sin B_m(k)\end{bmatrix} \tag{17}$$

$$X_v=\begin{bmatrix}\dot{x}_t\\ \dot{y}_t\end{bmatrix} \tag{18}$$

$$X_p(0)=\begin{bmatrix}R_x(0)\\ R_y(0)\end{bmatrix} \tag{19}$$

Let A=–A', then the least square of estimation of X_v is,

$$\hat{X}_v=\left[\left(A^TA\right)^{-1}A^TD\right]X_p(0) \tag{20}$$

$$\text{Let } S=\left[\left(A^TA\right)^{-1}A^TD\right] \tag{21}$$

Then, we can find out target course as follows,

$$\dot{x}_t=S_{11}\,R_x(0)+S_{12}\,R_y(0)$$

$$\dot{y}_t=S_{21}\,R_x(0)+S_{22}\,R_y(0)$$

Replacing $R_x(0)$ with $Tan(B_0)R_y(0)$ in the above equations, (where B_0 is the initial bearing measurement), we obtain,

$$\dot{x}_t=S_{11}\,Tan(B_0)\,R_y(0)+S_{12}\,R_y(0) \tag{22}$$

$$\dot{y}_t=S_{21}\,Tan(B_0)\,R_y(0)+S_{22}\,R_y(0)$$

And equation (22) can be written as,

$$\frac{\dot{x}_t}{\dot{y}_t}=\frac{S_{11}\,Tan(B_0)+S_{12}}{S_{21}\,Tan(B_0)+S_{22}} \tag{23}$$

$$Tan^{-1}\left(\frac{\dot{x}_t}{\dot{y}_t}\right)=Tan^{-1}\left(\frac{S_{11}\,Tan(B_0)+S_{12}}{S_{21}\,Tan(B_0)+S_{22}}\right) \tag{24}$$

The target course, tcr, can be calculated as,

$$tcr=Tan^{-1}\left(\frac{S_{11}\,Tan(B_0)+S_{12}}{S_{21}\,Tan(B_0)+S_{22}}\right) \tag{25}$$

Let us find out S matrix. For this, (A^TA) and (A^TD) are to be calculated.

$$A^TA=\begin{bmatrix}-t\cos B_m(1) & -2t.\cos B_m(2) & . & . & -kt.\cos B_m(k)\\ t.\sin B_m(1) & 2t.\sin B_m(2) & & . & kt.\sin B_m(k)\end{bmatrix}. \tag{26}$$

$$\begin{bmatrix}-t.\cos B_m(1) & t.\sin B_m(1)\\ -2t.\cos B_m(2) & 2t.\sin B_m(2)\\ . & .\\ . & .\\ -kt.\cos B_m(k) & kt.\sin B_m(k)\end{bmatrix}$$

$$=\begin{bmatrix}\sum_{i=1}^{k} i^2t^2\cos^2B_m(i) & \sum_{i=1}^{k} -i^2t^2\cos B_m(i)\sin B_m(i)\\ \sum_{i=1}^{k} -i^2t^2\cos B_m(i)\sin B_m(i) & \sum_{i=1}^{k} i^2t^2\sin^2B_m(i)\end{bmatrix} \tag{27}$$

If variance of the noise in the bearing is also considered, then,

$$(A^TA) = \begin{bmatrix} \sum_{i=1}^{k} \frac{i^2t^2\cos^2 B_m(i)}{\sigma^2(i)} & \sum_{i=1}^{k} \frac{-i^2t^2\cos B_m(i)\sin B_m(i)}{\sigma^2(i)} \\ \sum_{i=1}^{k} \frac{-i^2t^2\cos B_m(i)\sin B_m(i)}{\sigma^2(i)} & \sum_{i=1}^{k} \frac{i^2t^2\sin^2 B_m(i)}{\sigma^2(i)} \end{bmatrix} \quad (28)$$

Now consider A^TD,

$$A^TD = \begin{bmatrix} -t.\cos B_m(1) & -2t.\cos B_m(2) & . & . & -kt.\cos B_m(k) \\ t.\sin B_m(1) & 2t.\sin B_m(2) & . & . & kt.\sin B_m(k) \end{bmatrix} \quad (29)$$

$$\begin{bmatrix} \cos B_m(1) & -\sin B_m(k) \\ \cos B_m(2) & -\sin B_m(k) \\ . & . \\ . & . \\ \cos B_m(k) & -\sin B_m(k) \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{i=1}^{k} -i\,t\cos^2 B_m(i) & \sum_{i=1}^{k} i\,t\cos B_m(i)\sin B_m(i) \\ \sum_{i=1}^{k} i\,t\cos B_m(i)\sin B_m(i) & \sum_{i=1}^{k} -i\,t\sin^2 B_m(i) \end{bmatrix} \quad (30)$$

Considering the variance of noise in the measurements,

$$A^TD = \begin{bmatrix} \sum_{i=1}^{k} \frac{-i\,t\cos^2 B_m(i)}{\sigma^2(i)} & \sum_{i=1}^{k} \frac{i\,t\cos B_m(i)\sin B_m(i)}{\sigma^2(i)} \\ \sum_{i=1}^{k} \frac{i\,t\cos B_m(i)\sin B_m(i)}{\sigma^2(i)} & \sum_{i=1}^{k} \frac{-i\,t\sin^2 B_m(i)}{\sigma^2(i)} \end{bmatrix} \quad (31)$$

So far, the processing is in batch, in which the calculation time increases with number of samples. Let us find out target course in sequential processing [9].

Sequential processing

After the arrival of the first bearing measurement,

Let,

$$\text{ksums}[1] = \frac{\cos^2 B_m(1)}{\sigma^2(1)}$$

The sums are updated after the second measurement as follows,

$$\text{ksums}[1] = \frac{\cos^2 B_m(k)}{\sigma^2(k)} + \text{previous ksums}[1]$$

All the elements of equations (28) and (30) can be written as follows,

$$\text{ksums}[2] = \frac{-\cos B_m(k)\sin B_m(k)}{\sigma^2(k)} + \text{previous ksums}[2]$$

$$\text{ksums}[3] = \frac{\sin^2 B_m(k)}{\sigma^2(k)} + \text{previous ksums}[3]$$

$$\text{ksums}[4] = \frac{-kt.\cos B_m(k)\sin B_m(k)}{\sigma^2(k)} + \text{previous ksums}[4]$$

Where, k is the number of the measurement if the measurements are available continuously with sampling interval t seconds. If the measurements are not continuous, then kt represents the time elapsed so far from the beginning of the taking the measurements.

$$\text{ksums}[5] = \frac{kt.\cos^2 B_m(k)}{\sigma^2(k)} + \text{previous ksums}[5]$$

$$\text{ksums}[6] = \frac{kt.\sin^2 B_m(k)}{\sigma^2(k)} + \text{previous ksums}[6]$$

$$\text{ksums}[7] = \frac{k^2t^2.\cos^2 B_m(k)}{\sigma^2(k)} + \text{previous ksums}[7]$$

$$\text{ksums}[8] = \frac{-k^2t^2.\cos B_m(k)\sin B_m(k)}{\sigma^2(k)} + \text{previous ksums}[8]\,;$$

$$\text{ksums}[9] = \frac{k^2t^2.\cos^2 B_m(k)}{\sigma^2(k)} + \text{previous ksums}[9] \quad (32)$$

So, A^TA, equation (28) can be written as,

$$A^TA = \begin{bmatrix} \text{ksums}[7] & \text{ksums}[8] \\ \text{ksums}[8] & \text{ksums}[9] \end{bmatrix} \quad (33)$$

Similarly, A^TD, equation (31) can be written as,

$$A^TD = \begin{bmatrix} \text{ksums}[4] & \text{ksums}[5] \\ \text{ksums}[5] & \text{ksums}[6] \end{bmatrix} \quad (34)$$

In this way, the target course can be found out easily after 2nd measurement onward.

TMPs when initial range is known

Sometimes, range between the target and observer is available through some means. Then, the other parameters namely the target course and speed can be calculated as follows. Here onward, we can relax the constraint that ownship is not moving. Let the ownship be in its usual condition that it is moving. We can write that

$$X_s(1) = \begin{bmatrix} \dot{x}_t(1) \\ \dot{y}_t(1) \\ R_x(1) \\ R_y(1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ t & 0 & 1 & 0 \\ 0 & t & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x}_t(0) \\ \dot{y}_t(0) \\ R_x(0) \\ R_y(0) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -(x_0(1)-x_0(0)) \\ -(y_0(1)-y_0(0)) \end{bmatrix} = \Phi(1,0).X_s(0) + b(1) \quad (35)$$

Where,

$$b(1) = \begin{bmatrix} 0 & 0 & -\left(x_0(1)-x_0(0)\right) & -\left(y_0(1)-y_0(0)\right) \end{bmatrix}$$

Similarly, we can write equations for $X_s(2)$, $X_s(3)$, and so on. The equation (14) becomes,

$$\begin{bmatrix} t.\cos B_m(1) & -t.\sin B_m(1) & \cos B_m(1) & -\sin B_m(1) \\ 2.t.\cos B_m(2) & -2.t.\sin B_m(2) & \cos B_m(2) & -\sin B_m(2) \\ & . & & \\ & . & & \\ k.t.\cos B_m(k) & -k.t.\sin B_m(k) & \cos B_m(k) & -\sin B_m(k) \end{bmatrix} \begin{bmatrix} \dot{x}_t \\ \dot{y}_t \\ R_x(0) \\ R_y(0) \end{bmatrix}$$

$$- \begin{bmatrix} \cos B_m(1).\left(x_0(1)-x_0(0)\right) - \sin B_m(1).\left(y_0(1)-y_0(0)\right) \\ \cos B_m(2).\left(x_0(2)-x_0(0)\right) - \sin B_m(2).\left(y_0(2)-y_0(0)\right) \\ . \\ . \\ \cos B_m(k).\left(x_0(k)-x_0(0)\right) - \sin B_m(k).\left(y_0(k)-y_0(0)\right) \end{bmatrix} + N(k) = 0 \quad (36)$$

Equation (36) can be written as,

$$\begin{bmatrix} t.\cos B_m(1) & -t.\sin B_m(1) \\ 2.t.\cos B_m(2) & -2.t.\sin B_m(2) \\ . & . \\ . & . \\ k.t.\cos B_m(k) & -k.t.\sin B_m(k) \end{bmatrix} \begin{bmatrix} \dot{x}_t \\ \dot{y}_t \end{bmatrix} + \begin{bmatrix} \cos B_m(1) & -\sin B_m(1) \\ \cos B_m(2) & -\sin B_m(2) \\ . & . \\ . & . \\ \cos B_m(k) & -\sin B_m(k) \end{bmatrix} \begin{bmatrix} R_x(0) \\ R_y(0) \end{bmatrix}$$

$$- \begin{bmatrix} \cos B_m(1).(x_0(1)-x_0(0)) - \sin B_m(1).(y_0(1)-y_0(0)) \\ \cos B_m(2).(x_0(2)-x_0(0)) - \sin B_m(2).(y_0(2)-y_0(0)) \\ . \\ . \\ \cos B_m(k).(x_0(k)-x_0(0)) - \sin B_m(k).(y_0(k)-y_0(0)) \end{bmatrix} + N(k) = 0 \qquad (37)$$

The equation (37) can be written as (like equation [15])

$$A'X_v + DX_p(0) - C + N(k) = 0 \qquad (38)$$

Where,

$$C = \begin{bmatrix} \cos B_m(1).(x_0(1)-x_0(0)) - \sin B_m(1).(y_0(1)-y_0(0)) \\ \cos B_m(2).(x_0(2)-x_0(0)) - \sin B_m(2).(y_0(2)-y_0(0)) \\ . \\ . \\ \cos B_m(k).(x_0(k)-x_0(0)) - \sin B_m(k).(y_0(k)-y_0(0)) \end{bmatrix} \qquad (39)$$

The least square solution of equation (38) can be written as,

$$X_v = \left(A^T.A\right)^{-1}.A^T.\left(D.X_p - C\right) \qquad (40)$$

A^TA is already available, as in equation (28). Now, let us calculate $A^T\left(B.X_p - C\right)$

$$A^T.DX_p = \begin{bmatrix} \sum -i\ t\cos^2 B_m(i) & \sum i.t\sin B_m(i).\cos B_m(i) \\ \sum i.t\sin B_m(i).\cos B_m(i) & \sum -i.t\sin^2 B_m(i) \end{bmatrix} . \begin{bmatrix} R_x(0) \\ R_y(0) \end{bmatrix}$$

$$= \begin{bmatrix} \sum -i.t\cos^2 B_m(i).R_x(0) + \sum i.t.\sin B_m(i).\cos B_m(i).R_x(0) \\ \sum i.t\cos B_m(i).\sin B_m(i).R_x(0) + \sum -i.t\sin^2 B_m(i).R_y(0) \end{bmatrix} \qquad (41)$$

$$A^T.D.X_p - A^T.C \qquad (42)$$

$$= \begin{bmatrix} \sum -i.t\cos^2 B_m(i).R_x(0) + \sum i.t\sin B_m(i).\cos B_m(i).R_y(0) + \\ \sum i.t\cos B_m(i) \begin{bmatrix} \cos B_m(i).(x_0(i)-x_0(0)) \\ -\sin B_m(i).(y_0(i)-y_0(0)) \end{bmatrix} \\ \sum i.t\sin B_m(i).\cos B_m(i).R_y(0) + \sum -i.t\sin^2 B_m(i).R_y(0) - \end{bmatrix}$$

$$\sum i.t\sin B_m(i) \begin{bmatrix} \cos B_m(i).(x_0(i)-x_0(0)) \\ -\sin B_m(i).(y_0(i)-y_0(0)) \end{bmatrix}$$

This is further simplified as,

$A^TDX_p - A^T.C$

$$= \begin{bmatrix} \sum -i.t\cos^2 B_m(i).R_x(0) + \sum i.t.\sin B_m(i).\cos B_m(i).R_y(0) + \\ \sum i.t.\cos B_m(i)\ [\text{term}(i)] \\ \sum i.t.\sin B_m(i).\cos B_m(i).R_x(0) + \sum -i.t\sin^2 B_m(i).R_y(0) - \\ \sum i.t.\sin B_m(i)\ [\text{term}(i)] \end{bmatrix} \qquad (43)$$

Where, $\text{term}(i) = \left[\cos B_m(i).(x_0(i)-x_0(0)) - \sin B_m(i).(y_0(i)-y_0(0))\right]$ (44)

For simplification of the process, equation (40) is modified as shown below.

$$\begin{bmatrix} \left(\dfrac{\dot{x}}{R_{y0}}\right) \\ \left(\dfrac{\dot{y}}{R_{y0}}\right) \end{bmatrix} = (A^TA)^{-1} \qquad (45)$$

$$\begin{bmatrix} \sum -i.t.\cos^2 B_m(i).\tan(B_0) + \\ \sum i.t.\sin B_m(i).\cos B_m(i) + \left(\dfrac{\sum i.t.\cos B_m(i).\text{term}(i)}{R_y(0)}\right) \\ \sum i.t.\sin B_m(i).\cos B_m(i).\tan(B_0) + \\ \sum -i.t.\sin^2 B_m(i) - \left(\dfrac{\sum i.t.\sin B_m(i).\text{term}(i)}{R_y(0)}\right) \end{bmatrix}$$

Sequential processing

As we have done earlier, let us convert equation (45) for sequential processing. Let,

$$\begin{aligned} &\text{ksums4\_mod} = -k.t.\cos^2 B_m(k)\ \tan(B_0) + \text{previous7\_mod} \\ &\text{ksums5\_mod} = k.t.\cos B_m(k)\ .\sin B_m(k) + \text{previous5\_mod} \\ &\text{ksums6\_mod} = -k.t.\sin^2 B_m(k) + \text{previous6\_mod} \\ &\text{ksums5\_mod} = k.t\ \cos B_m(k).\sin B_m(k)\tan(B_0) + \text{previous5\_mod} \\ &\text{term}(k) = \cos B_m(k)\left[x_0(k) - x_0(0)\right] - \sin B_m(k)\left[y_0(k) - y_0(0)\right] \\ &\text{lsums1} = k.t.\cos B_m(k) *\ \text{term}(k) + \text{previous lsums}[1] \\ &\text{lsums2} = -k.ts.\sin B_m(k) *\ \text{term}(k) + \text{previousLsums}[2] \\ &\text{lsums1\_mod} = \text{lsums1}/R_y(0) \\ &\text{lsums2\_mod} = \text{lsums2}/R_y(0) \end{aligned} \qquad (46)$$

Let us assume that we know initial range that is,

$R_y(0)$=initial range * cos $B_m(0)$

Using the above, we can write that

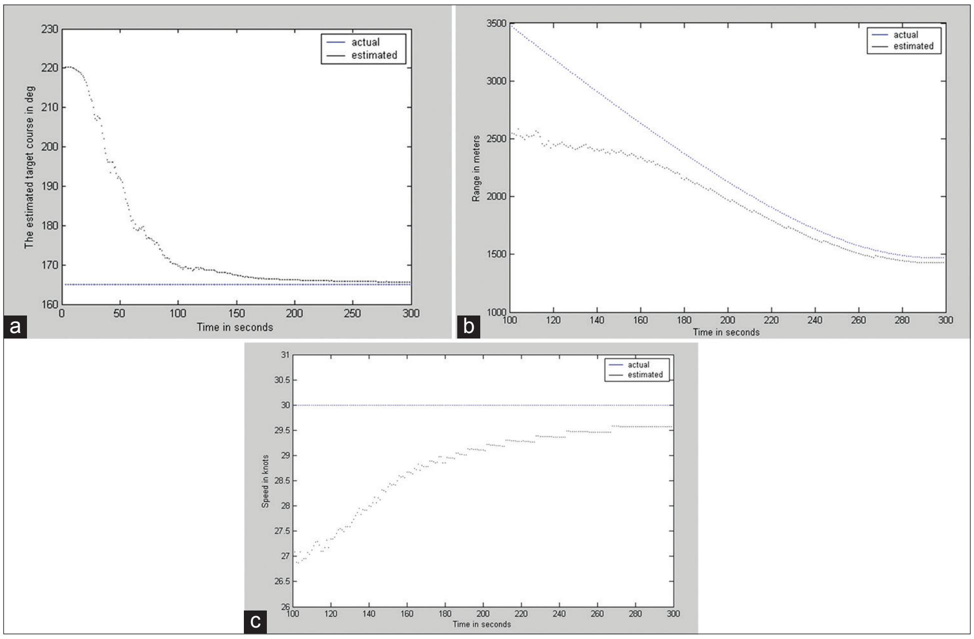

Fig. 1: (a) Error in course estimate, (b) error in range estimate, (c) error in speed estimate

$$A^T DX_p - A^T C = \begin{bmatrix} \text{ksums4\_mod} + \text{ksums5} + \text{Lsums1\_mod} \\ \text{ksums5\_mod} + \text{ksums6} + \text{Lsums2\_mod} \end{bmatrix} \quad (47)$$

and

$$(A^T A)^{-1} * (A^T DX_p - A^T C) =$$

$$\begin{bmatrix} \text{ksums7} & \text{ksums8} \\ \text{ksums8} & \text{ksums9} \end{bmatrix}^{-1} \begin{bmatrix} \text{ksums4\_mod} + \text{ksums5} + \text{Lsums1\_mod} \\ \text{ksums5\_mod} + \text{ksums6} + \text{Lsums2\_mod} \end{bmatrix} = [Q] \quad (48)$$

The target course (tcr), can be calculated as,

$$\text{tcr} = \text{Tan}^{-1}\left(\frac{Q\,(1,1)}{Q\,(2,1)}\right) \quad (49)$$

The target speed, vt, can be calculated as,

$$\text{vt} = \sqrt{Q\,(1,1)^2 + Q\,(2,1)^2} * R_y(0) \quad (50)$$

TMPs when target course/speed is known

Sometimes, the target course or speed is available through some means. Then, the other parameters can be calculated as follows, using equation (49) or (50). Let the target course is known. For several of values assumed initial range, find out the target courses. Let these are called calculated target courses. That range for which calculated target course is equal to the actual target course is the estimated target range. Once the range is known, we can calculate easily target speed. Similar procedure holds good when the target speed is known (This trial and error method is better than finding out transcendental explicit equation using Equation (45) for range estimation. These days very high-speed processors are available, and hence, the above suggested trail and method generates the solution fast).

SIMULATION AND RESULTS

The purpose of this paper is to find out the TMPs by some means, but without asking for ownship to maneuver, which is a very troublesome affair many times to the operator of the vehicle. There may be several methods to achieve this goal. Here, I am suggesting one method that is to obtain the target course, by restricting the underwater vehicle to be in standstill position for some time. Once target course is found out, the ownship can move. Then, using the target course, the other parameters such as target speed and range can be found out. Provision is given in the algorithm, to find out TMPs, if range/speed of the target is known (Then, the ownship need not be in standstill condition and to obtain the target course. The vehicle can be moving from the beginning of the processes).

The algorithm is realized using Matlab, which is realizable on any personal computer. All one-second samples are corrupted by additive zero mean Gaussian noise with max 0.5°. It is also assumed that the bearing measurements are available continuously every second. The simulation is carried out by considering the target at an initial range of 5000 m with 40° initial bearing relative to the observer. The target is assumed to be moving at a speed of 30 knots (15.45 m/s). The ownship is assumed to be moving at a speed of 20 knots (10.3 m/s) (at zero speed while estimating the target course) and at 90°course. (All angels are considered with respect to Y axis). Number of scenarios is tested by changing the course of the target in steps of 1° in such a way that the angle between the target course and line of sight is always <55°, as only closing targets are of interest to the observer. In general, the error allowed in the estimated TMPs in underwater are 8% in range estimate, 0.2° in bearing estimate, 5° in course estimate, and 3 m/s in velocity. For the purpose of analysis, this scenario with target course equal to 165° is considered. The results of this scenario after several Monte Carlo runs are shown in Fig. 1. From the results, it is observed that course estimate with required accuracy is obtained from around 100 seconds onward. For the same scenario, the target course is fed from 100 seconds onward to estimate the range and speed. The results are shown in Fig. 1b and c.

SUMMARY AND CONCLUSION

The method introduced by Chan and Rea [5] is implemented in sequential processing. Here, recursive SUMS are introduced and updated whenever a new bearing measurement is available. The constraint here is while estimating the target course; the ownship has to be standstill condition. From these results, it is concluded that this algorithm can be utilized for passive target tracking applications.

REFERENCES

1. Jaffret C, Bar-Shalom Y. Track formation with bearing and frequency measurements. IEEE Trans Aerosp Electron Syst 1990;26:999-1009.
2. Chan YT, Rudnicki SW. Bearings-only and doppler-bearing tracking

using instrumental variables. IEEE Trans Aeros Electron Syst 1992;28:1076-82.

3. Xiuo-Jiao T, Cai-Rong Z, Zhen-Ya H. Passive target tracking using maximum likelihood estimation. IEEE Trans Aeros Electron Syst 1996;32:1348-53.
4. Song TL, Speyer JL. A stochastic analysis of a modified gain extended Kalman filter with applications to estimation with bearing only measurements. IEEE Transactions on Automat Contr 1985;AC-30(10):940-9.
5. Chan YT, Rea TA. Passive tracking scheme for a single stationary observer. IEEE Trans Aerosp Electron Syst 2002;38(3):1046-53.
6. Lindgren AG, Gong KF. Position and velocity estimation via bearing observations. IEEE Trans Aerosp Electron Syst 1978;AES-14:564-77.
7. Lindgren AG, Nardone SC. Biased estimation properties of the pseudo linear estimator. IEEE Trans Aerosp Electron Syst 1982;AES-18(4):432-41.
8. Nardone SC, Lindgren AG, Gong KF. Fundamental properties and performance of conventional bearings only target motion analysis. IEEE Trans Automat Contr 1984;Ac-29(9):775-87.
9. Koteswara Rao S. Pseudo-linear estimator for bearings-only passive target tracking. IEE Proceedings-Radar, Sonar, Navigation. Vol. 148. p. 16-22.