

AN UNSTEADY MAGNETO HYDRODYNAMIC HEAT TRANSFER FLOW IN A ROTATING PARALLEL PLATE CHANNEL THROUGH A POROUS MEDIUM WITH RADIATION EFFECT

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Received: 16 September 2016, Revised and Accepted: 24 September 2016

ABSTRACT

Objective: The combined effect of a transverse magnetic field and radiative heat transfer on unsteady flow of a conducting optically thin viscoelastic fluid through a rotating channel filled with saturated porous medium and nonuniform walls temperature has been discussed.

Methods: Supposed that the fluid has minor electrical conductivity and the electromagnetic power taken as small. The set of equations is solved by perturbation technique.

Results: The flow governed by the nondimensional parameters, α is the viscoelastic parameter, R is the radiation parameter with fixed values of Gr the Grashoff number, and Pe Peclet parameter. The expressions for velocity, temperature, and species concentration fields are obtained. The effects of numerous physical parameters on the above flow quantities are studied with the help of graphs.

Conclusions: The magnitude of the velocity component u experiences retardation and the behaviors of the velocity component v remain the same with the increasing values of the Hartmann number. The magnitude of the temperature increases with increasing Re , D , α , R , E , and experiences retardation with increasing the magnetic field parameter. Due to brevity, more content will not be presented.

Keywords: Radiation effects, Heat transfer, Magnetohydrodynamics flows, Porous medium.

INTRODUCTION

Heat transfer in porous media has received considerable attention and has been the field of a number of investigations during the last two decade. The need for fundamental studies in porous media heat transfer stems from the fact that a better understanding of a host of thermal engineering applications, in which porous materials are present is required. Some of the examples of thermal engineering disciplines which stand to benefit from a better understanding of heat and fluid flow processes through porous media are geothermal systems, thermal insulations, grain storage, solid matrix heat exchangers, oil extraction, and manufacturing numerous products in the chemical industry. The application of electromagnetic fields in controlling the heat transfer as in aerodynamic heating leads to the study of magnetohydrodynamic heat transfer. This magnetohydrodynamics (MHD) heat transfer has gained significance owing to recent advancement of space technology. The MHD heat transfer can be divided into two parts. One contains problems, in which the heating is incidental by-product of electromagnetic fields as in MHD generators and pumps, etc., and the second consists of problems, in which the primary use of electromagnetic fields is to control the heat transfer. Heat transfer in channels partially filled with porous media has gained considerable attention in recent years because of its various applications in contemporary technology. These applications include porous journal bearing, blood flow in lungs or in arteries, nuclear reactors, porous flat plate collectors, packed bed thermal storage solidification of concentrated alloys, fibrous and granular insulation, grain storage and drying, paper drying, and food storage. Besides, the use of porous substrates to improve heat transfer in channels, which is considered as porous layers, finds applications in heat exchangers, electronic cooling, heat pipes, filtration and chemical reactors, etc. In these applications, engineers avoid filling entire channel with a solid matrix to reduce the pressure drop. The flow between parallel plates is a classical problem that has important applications in magneto hydrodynamic power generators and pumps, accelerators, aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil, and fluid droplets and sprays. The stream of an electrically directing liquid has essential

applications in numerous branches of designing science. A review of MHD studies in the mechanical fields can be found in Moreau [1]. The stream of liquids through permeable media is a critical subject due to the recuperation of raw petroleum from the pores of the store rocks; for this situation, Darcy's law speaks to the gross impact. Raptis *et al.* [2] have explained the hydro attractive free convection course through a permeable medium between two parallel plates. Aldoss *et al.* [3] have discussed on blended convection stream from a vertical plate inserted in a permeable medium within sight of an attractive field. Makinde and Mhone [4] have considered heat transfer to MHD oscillatory stream in a channel loaded with the permeable medium. The outcomes are great concurrence with [4]. The consolidated impact of a transverse attractive field and radiative heat transfer on insecure stream of a leading optically thin viscoelastic liquid through a pivoting channel loaded with immersed permeable medium and nonuniform dividers temperature has been examined. The scientific arrangements are acquired through annoyance system. The constitutive condition for the incompressible second request liquid is of the structure.

$$\sigma = pI + \mu_1 A_1 + \mu_2 A_2 + \mu_3 (A_1)^2 \quad (1)$$

Where, σ is the stress tensor, p is the hydrostatic pressure, I is the unit tensor, and A_n ($n=1, 2$) are the kinematic Rivlin-Ericksen tensors, μ_1 , μ_2 , and μ_3 are the material coefficients describing viscosity, elasticity, and cross-viscosity, respectively.

The material coefficients μ_1 , μ_2 , and μ_3 have taken constants with μ_1 and μ_3 as positive and μ_2 as negative [5]. The Equation 1 was derived by Coleman and Noll [6] from that of the simple fluids by assuming that stress is more sensitive to the recent deformation than to the deformation that occurred in the distant past.

Mathematical formulation and solution of the problem

Consider the flow of a conducting optically thin fluid in a channel filled with saturated porous medium under the influence of an externally applied homogeneous magnetic field and radiative heat transfer. It is assumed that the fluid has small electrical conductivity and the

electromagnetic force produced is very small. The x-axis is taken along the center of the channel, and the z-axis is taken normal to it. In the initial undisturbed state, both the plates and the fluid rotate with the same angular velocity Ω . At $t > 0$, the fluid is driven by a constant pressure gradient parallel to the channel walls. Then, assuming a Boussinesq incompressible fluid model, the equations governing the motion are given by,

$$\frac{\partial u}{\partial t} - 2\Omega v = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu_1 \frac{\partial^2 u}{\partial z^2} + \tag{2}$$

$$\nu_2 \frac{\partial^3 u}{\partial z^2 \partial t} - \frac{\sigma_e B_0^2 u}{\rho} - \nu_1 \frac{u}{K} + g\beta(T - T_0)$$

$$\frac{\partial v}{\partial t} + 2\Omega u = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu_1 \frac{\partial^2 v}{\partial z^2} + \tag{3}$$

$$\nu_2 \frac{\partial^3 v}{\partial z^2 \partial t} - \frac{\tilde{A}_e B_0^2 v}{\rho} - \nu_1 \frac{v}{K}$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial z^2} - \frac{1}{\rho C_p} \frac{\partial q_1}{\partial z} \tag{4}$$

Corresponding boundary conditions:

$$u=0, v=0, T=T_w$$

$$\text{on } z=1. \tag{5}$$

$$u=0, v=0, T=T_0$$

$$\text{on } z=0 \tag{6}$$

It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small.

Where, $B_0 (= \mu_e H_0)$ and $\nu_i = \mu_i / \rho$, ($i=1, 2$). It is assumed that both walls of temperature T_w, T_0 are high enough to induce radiative heat transfer. Following Cogley *et al.* [7], it is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by,

$$\frac{\partial q_1}{\partial z} = 4\alpha_1^2 (T_0 - T), \tag{7}$$

Combining Equations 2 and 3 and let $q = u + iv$ and $\xi = x + iy$, we obtain,

$$\frac{\partial q}{\partial t} + 2i\Omega q = -\frac{1}{\rho} \frac{\partial p}{\partial \xi} + \nu_1 \frac{\partial^2 q}{\partial z^2} + \nu_2 \frac{\partial^3 q}{\partial z^2 \partial t} \tag{8}$$

$$- \frac{\sigma_e B_0^2 q}{\rho} - \nu_1 \frac{q}{K} + g\beta(T - T_0)$$

The following nondimensional quantities are introduced:

$$x^* = \frac{x}{a}, y^* = \frac{y}{a}, \xi^* = \frac{\xi}{a}, z^* = \frac{z}{a},$$

$$u^* = \frac{u}{U}, v^* = \frac{v}{U}, q^* = \frac{q}{U},$$

$$t^* = \frac{tU}{a}, p^* = \frac{ap}{\rho \nu_1 U}, \theta = \frac{T - T_0}{T_w - T_0}$$

Making utilization of nondimensional variables, the dimensionless administering conditions together with suitable limit conditions (dropping indicators) are,

$$Re \frac{\partial q}{\partial t} = -\frac{\partial p}{\partial \xi} + \frac{\partial^2 q}{\partial z^2} + \alpha \frac{\partial^3 q}{\partial z^2 \partial t} \tag{9}$$

$$-(M^2 + 2iE^{-1} + S^2)q + GrT$$

$$Pe \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial z^2} + R^2 \theta \tag{10}$$

With,

$$q=0, \theta=1 \text{ on } z=1 \tag{11}$$

$$q=0, \theta=0 \text{ on } z=0 \tag{12}$$

Where,

$$Re = \frac{Ua}{\nu_1}, M^2 = \frac{\sigma_e \beta_0^2 a^2}{\nu_1 U}, E = \frac{\nu_1 U}{\Omega a^2}, D = \frac{k}{a^2}, S = \frac{1}{D}, \alpha = \frac{\nu_2 Re}{a^2},$$

$$Gr = \frac{g\beta(T_w - T_0)a^2}{\nu_1 U},$$

$$Pe = \frac{Ua\rho C_p}{k}, R^2 = \frac{4\alpha_1^2 a^2}{k}$$

Solving the Equations 9 and 10 for purely oscillatory flow, Let,

$$-\frac{\partial P}{\partial x} = \lambda e^{i\omega t}, q(z,t) = q_0(z) e^{i\omega t}, \tag{13}$$

$$\theta(z,t) = \theta_0(z) e^{i\omega t}$$

Where, λ is constant and ω is the frequency of oscillation.

Substituting the above expressions (13) into the Equations 9 and 10, and making use of the corresponding boundary conditions (11) and (12), we obtain,

$$(1 + i\alpha\omega) \frac{d^2 q_0}{dz^2} - m_1^2 q_0 = -\lambda - Gr\theta_0 \tag{14}$$

$$\frac{d^2 \theta_0}{dz^2} + m_2^2 \theta_0 = 0 \tag{15}$$

Subjected to the boundary conditions,

$$q_0=0, \theta_0=1 \text{ on } z=1 \tag{16}$$

$$q_0=0, \theta_0=0 \text{ on } z=0 \tag{17}$$

$$\text{Where, } m_1 = \sqrt{M^2 + 2iE^{-1} + S^2 + i\omega Re} \text{ and } m_2 = \sqrt{R^2 - i\omega Pe}$$

Equations 14 and 15 are solved.

We obtained the solution for the fluid velocity and temperature as follows:

$$q(z,t) = \left\{ a_1 e^{(m_1 z)/b} + \left(a_1 - \frac{\lambda}{m_1^2} \right) e^{-(m_1 z)/b} + \frac{\lambda}{m_1^2} + \frac{Gr \sin(m_2 z)}{(m_2^2 b + m_1^2) \sin(m_2)} \right\} e^{i\omega t} \tag{18}$$

$$\theta(z,t) = \frac{\sin(m_2 z)}{\sin(m_2)} e^{i\omega t} \tag{19}$$

$$\text{Where, } a_1 = \frac{\left(\left(\frac{\lambda}{m_1^2} \right) e^{-m_1/b} - \left(\frac{\lambda}{m_1^2} \right) - \frac{Gr}{(m_2^2 b + m_1^2)} \right)}{\left(e^{m_1/b} - e^{-m_1/b} \right)}, b = 1 + i\alpha\omega$$

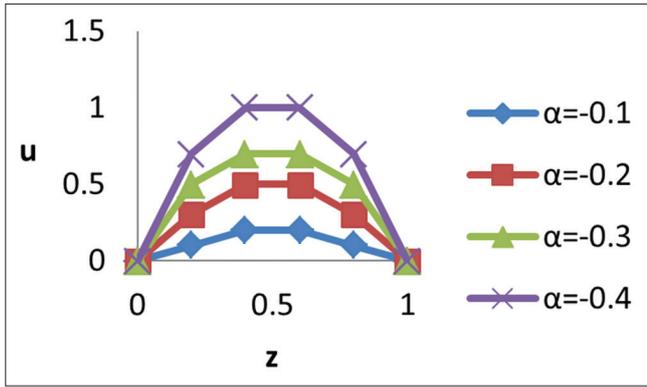


Fig. 1: Velocity profile for u on visco-elastic parameter

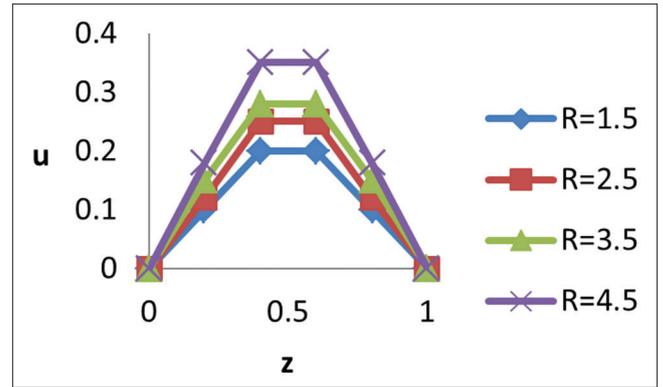


Fig. 4: Velocity profile for v on Radiation parameter

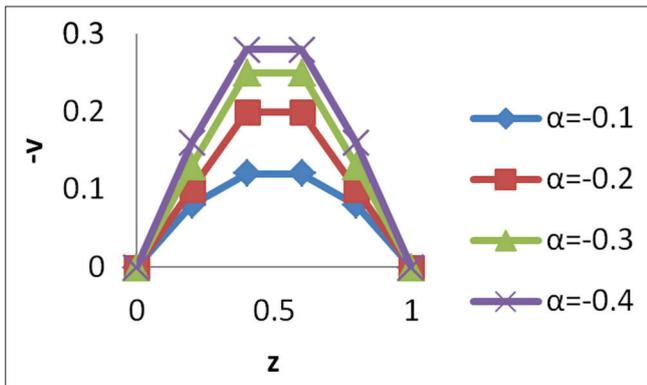


Fig. 2: Velocity profile for u on visco-elastic parameter

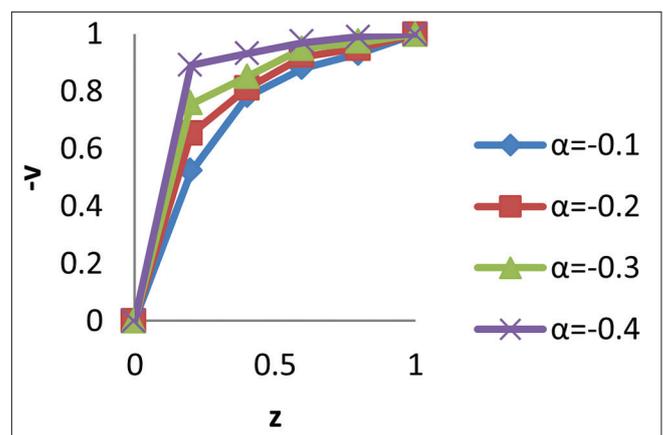


Fig. 5: Temperature profile for v on visco-elastic parameter

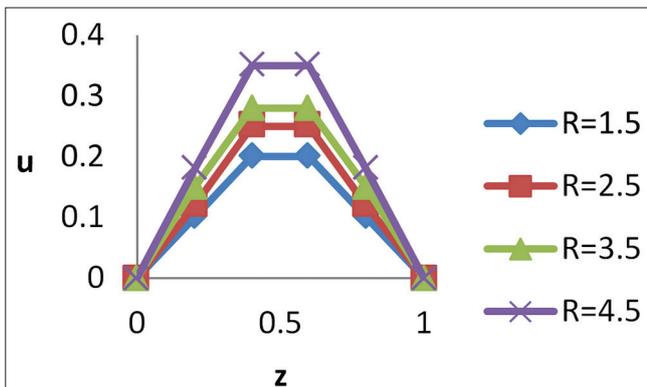


Fig. 3: Velocity profile for u on Radiation parameter

RESULTS AND DISCUSSION

The stream represented by the nondimensional parameters, α is the viscoelastic parameter, and R is the Radiation parameter with altered qualities. We have considered the genuine and nonexistent parts of the outcomes u and v all through the problem. The velocity profiles for the segments against z is plotted in Figs. 1-4, whereas Fig. 5 to watch temperature profiles on the viscoelastic impacts and Radiation parameters for different arrangements of qualities with $Pe = 2, t = 0.1, Gr = 2, \lambda = 1,$ and $\omega = 1$. It is apparent from (Figs. 1-4) that, the velocity profiles are explanatory in nature, and the extent of speed u and v increment with the expanding estimations of the viscoelastic parameter $|\alpha|$, radiation parameter R . We watch that lower the porousness of the permeable medium lesser the liquid pace in the whole liquid district. The resultant speed q improves with expanding the parameters $Re, D, |\alpha|, R,$ and encounters

impediment with expanding the force of the attractive field. It is apparent that the temperature profiles display the way of the stream on administering parameters. The extent of the temperature increments with expanding $Re, D, |\alpha|, R, E,$ and encounters hindrance with expanding the attractive field parameter (Hartmann number M).

It has additionally been watched that the temperature field is not essentially influenced by the viscoelastic parameter. The graphs were drawn with $Re = 50, M = 2, S = 1, E = 0.01, R = 1.5, \alpha = -0.1$.

CONCLUSIONS

The combined effect of a transverse magnetic field and radiative heat transfer on unsteady flow of a conducting optically thin viscoelastic fluid through a rotating channel filled with saturated porous medium and nonuniform walls temperature has been discussed. It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. The analytical solutions are obtained for the problem making use of perturbation technique.

1. The magnitude of velocity u and v increase with the increasing values of the viscoelastic parameter α , radiation parameter R .
2. The magnitude of the velocity component u experiences retardation and the behaviors of the velocity component v remain the same with the increasing values of M .
3. Lower the permeability of the porous medium lesser the fluid speed in the entire fluid region.

ACKNOWLEDGMENT

The author is very much grateful to the reviewers for their constructive and valuable suggestions for further improvement in this paper.

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