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Research Article

ON INTUITIONISTIC FUZZY BI-IDEALS IN GAMMA NEAR RINGS

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ABSTRACT

The fuzzy set theory developed by Zadeh and others has found many applications in the domain of mathematics. Gamma near-rings were defined by Satyanarayana and the ideal theory in gamma near rings was studied by Satyanarayana and Booth. In this paper, we introduce the intuitionistic fuzzy bi-ideals in Γ-near-rings and investigate some of their related properties.

Keywords: Γ-near-rings, Intuitionistic fuzzy ideals, Intuitionistic fuzzy bi-ideals.

1. INTRODUCTION

Following the introduction of fuzzy sets by Zadeh [20], the fuzzy set theory has been used for many applications in the domain of mathematics and elsewhere. The idea of "Intuitionistic Fuzzy Set (IFS)" was first published by Atanassov [1] as a generalization of the notion of fuzzy set. The concept of Γ-near-ring, a generalization of both the concepts nearing and Γ-ring, was introduced by Satyanarayana [16,17]. Later, several authors such as Booth and Satyanarayana [2,3,5-7,10-14,19] studied the ideal theory of Γ-near-rings. Later Jun *et al*. [8,9] considered the fuzzification of left (resp. right) ideals of Γ-near-rings. In this paper, we introduce the notion an intuitionistic fuzzy bi-ideal in a Γ-near-ring and some properties of such bi-ideals are investigated. The homomorphic property of intuitionistic fuzzy bi-ideals is established.

2. PRELIMINARIES

In this section, we include some elementary aspects that are necessary for this paper.

Definition 2.1 [16]: A nonempty set R with two binary operations "+" (addition) and "." (multiplication) is called a near-ring if it satisfies the following axioms:
i. (R, +) is a group,
ii. (R,.) is a semigroup,
iii. $(x + y).z = x.z + y.z$, for all $x, y, z \in R$. It is a right near-ring because it satisfies the right distributive law.

Definition 2.2 [17]: A Γ-near-ring is a triple (M, +, Γ) where,
i. (M, +) is a group,
ii. Γ is a nonempty set of binary operators on M such that for each $\alpha \in \Gamma$, (M, +, α) is a near-ring,
iii. $x\,\alpha\,(y\,\beta\,z) = (x\,\alpha\,y)\beta\,z$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

Definition 2.3 [17]: A subset A of a Γ-near-ring M is called a left (resp. right) ideal of M if
i. (A, +) is a normal divisor of (M, +),
ii. $u\,\alpha\,(x + v) - u\alpha v \in A$ (resp. $x\,\alpha\,u \in A$) for all $x \in A, \alpha \in \Gamma$ and $u, v \in M$.

Definition 2.4 [18]: Let M be Γ-near-ring. A subgroup A of M is called a bi-ideal of M if $(A\Gamma M\Gamma A) \cap (A\Gamma M)\,\Gamma^*A \subseteq A$. where the operation "*" is defined by,

$A\Gamma^*B = \{a\gamma(a' + b) - a\gamma a')/a, a' \in A, \gamma \in \Gamma, b \in B\}$.

Definition 2.5 [17]: Let M be Γ-near-ring. A subgroup Q of M is called a quasi-ideal of M if $(Q\Gamma M) \cap (M\Gamma Q) \cap (M\Gamma)^*Q \subseteq Q$.

Definition 2.6 [9]: Let M and N be Γ-near-rings. A mapping f: M → N is said to be a homomorphism if $f(a\,\alpha\,b) = f(a)\,\alpha\,f(b)$ for all $a, b \in M$ and $\alpha \in \Gamma$.

Definition 2.7 [9]: A fuzzy set μ in a Γ-near-ring M is called a fuzzy left (resp. right) ideal of M if,
i. $\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$,
ii. $\mu(y + x-y) \geq \mu(x)$, for all $x, y \in M$,
iii. $\mu(u\,\alpha\,(x + v) - u\,\alpha\,v) \geq \mu(x)$ (resp. $\mu(x\,\alpha\,u) \geq \mu(x)$) for all $x, u, v \in M$ and $\alpha \in \Gamma$.

Definition 2.9 [1]: Let X be a nonempty fixed set. An IFS A in X is an object having the form $A = \{< x, \mu_A(x), \nu_A(x) >/x \in X\}$, where the functions $\mu_A: X \to [0, 1]$ and $\nu_A: X \to [0, 1]$ denote the degree of membership and degree of nonmembership of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Notation: For the sake of simplicity, we shall use the symbol $A= <\mu_A, \nu_A>$ for the IFS,

$A = \{<x, \mu_A(x), \nu_A(x) >/x \in X\}$.

Definition 2.10 [1]: Let X be a nonempty set and let $A = <\mu_A, \nu_A> \mu\nu$ and $B = <\mu_B, \nu_B>$ be IFSs in X. Then:
1. $A \subset B$ if $\mu_A \leq \mu_B$ and $\nu_A \geq \nu_B$
2. $A = B$ if $A \subset B$ and $B \subset A$
3. $A^c = <\nu_A, \mu_A>$
4. $A \cap B = (\mu_A \wedge \mu_B, \nu_A \vee \nu_B)$
5. $A \cup B = (\mu_A \vee \mu_B, \nu_A \wedge \nu_B)$
6. $\Box A = (\mu_A, 1-\mu_Q)$, $\Diamond A = (1-\nu_A, \nu_A)$

Definition 2.11 [14]: Let A be an IFS in a Γ-near-ring M. For each pair <τ, σ> ϵ [0, 1] ∈with $t + s \leq 1$, the set $A_{<t, s>} = \{x \in X/\mu_A(x) \geq t$ and $\nu\nu_A(x) \leq s\}$ is called a <t, s> level subset of A.

Definition 2.12 [14]: Let $A = <\mu_A,\nu_A>\mu\nu$ be an IFS in M and let $t \in [0, 1]$. Then the sets $U(\mu_A;t) = \{x\in M: \mu_A(x) \geq t\}$ and $L\nu(\nu_A; t) = \{x\in M: \nu_A(x) \leq t\}$ are called upper level set and lower level set of A, respectively.

3. INTUITIONISTIC FUZZY BI-IDEALS OF Γ-NEAR-RINGS

In what follows, M will denote a Γ-near-ring unless otherwise specified.

Definition 3.1: An intuitionistic fuzzy ideal $A = <\mu A, \nu_A>$ of M is called an intuitionistic fuzzy bi-ideal of M if,

i. $\mu_A(x-y) \geq \{\mu_A(x) \wedge \mu_A(y)\}$,
ii. $\mu_A(y + x-y) \geq \mu_A(x)$,
iii. $\mu_A((x\ \alpha\ y\ \beta\ z) \wedge (x\ \alpha\ (y + z) - x\ \alpha\ z)) \geq \{\mu_A(x) \wedge \mu_A(z)\}$for all $x, y, z \in M$, $\alpha, \beta \in \Gamma$.
iv. $\nu_A(x-y) \leq \{\nu_A(x) \vee \nu_A(y)\}$,
v. $\nu_A(y + x-y) \leq \nu_A(x)$,
vi. $\nu_A((x\ \alpha\ y\ \beta\ z) \vee (x\ \alpha\ (y + z) - x\ \alpha\ z)) \leq \nu\{\nu_A(x) \vee \nu_A(z)\}$ for all $x, y, z \in M$, $\alpha, \beta \in \Gamma$.

Example 3.2.: Let R be the set of all integers then R is a ring. Take $M = \Gamma = R$. Let $a, b \in M$, $\alpha \in \Gamma$, suppose $a\alpha b$ is the product of $a, \alpha, b \in R$.

Then M is a Γ–near-ring.
Define an IFS $A = <\mu_A, \nu_A>\mu\nu$ in R as follows.
$\mu_A(0) = 1$ and $\mu_A(\pm 1) = \mu_A(\pm 2) = \pm\mu_A(\pm 3) = ... = t$ and
$\nu_A(0) = 0$ and $\pm\nu_A(\pm 1) = \nu_A(\pm 2) = \nu_A(\pm 3) = ... = s$,
Where, $t \in [0, 1)$, $s \in (0, 1]$ and $t + s \leq 1$.

By routine calculations,
Clearly, A is an intuitionistic fuzzy bi-ideal of a Γ-near-ring M.

Lemma 3.3.: If B is a bi-ideal of M then for any $0 < t, s < 1$, there exists an intuitionistic fuzzy bi-ideal $C = < \mu_C, \nu_C >$ of M such that $C_{<t, s>} = B$.

Proof: Let $C \to [0, 1]$ be a function defined by

$$\mu_B(x) = \begin{cases} t & \text{if } x \in B, \\ 0 & \text{if } x \notin B, \end{cases} \qquad \nu_B(x) = \begin{cases} s & \text{if } y \in B, \\ 1 & \text{if } y \notin B. \end{cases}$$

For all $x \in M$ and the pair $s, t \in [0, 1]$. Then $C_{<t, s>} = B$ is an intuitionistic fuzzy bi-ideal of M with $t + s \leq 1$.

Now suppose that B is a bi-ideal of M. For all $x, y \in B$, such that, $x-y \in B$, We have,

$\mu_c(x-y) \geq t = \{\mu_c(x) \wedge \mu_c(y)\}$,
$\nu_c(x-y) \leq s = \{\nu_c(x) \vee \nu_c(y)\}$,
$\mu_c(y + x-y) \geq t = \mu_c(x)$,
$\nu_c(y + x-y) \leq s = \nu_c(x)$,

Also, for all $x, y, z \in B$ and $\alpha, \beta \in \Gamma$ such that $x\alpha y\beta z \in B$, we have,
$\mu_c((x\ \alpha\ y\ \beta\ z) \wedge (x\ \alpha\ (y + z) - x\ \alpha\ z))) \geq t = \{\mu_c(x) \wedge \mu_c(z)\}$,
$\nu_c((x\ \alpha\ y\ \beta\ z) \vee (x\ \alpha\ (y + z) -x\ \alpha\ z))) \leq s = \{\nu_c(x) \vee \nu_c(z)\}$.

Thus $C_{<t, s>}$ is an intuitionistic fuzzy bi-ideal of M.

Lemma 3.4.: Let B be a nonempty subset of M. Then B is a bi-ideal of M if and only if the IFS $\overline{B} = <\chi_B, \overline{\chi}_B >$ is an intuitionistic fuzzy ideal of M.

Proof: Let $x, y \in B$. From the hypothesis, $x-y \in B$.

i. If $x, y \in B$, then $\chi_B(x) = 1$, $\overline{\chi}_B(x) = 0$, $\chi_B(y) = 1$ and $\overline{\chi}_B(y) = 0$. In this case,
$\chi_B(x-y) = 1 \geq \{\chi_B(x) \wedge \chi_B(y)\}$.
$\overline{\chi}_B(x-y) = 0 \leq \{\overline{\chi}_B(x) \vee \overline{\chi}_B(y)\}$.
ii. If $x \in B$, $y \notin B$, then $\chi_B(x) = 1$, $\overline{\chi}_B(x) = 0$, $\chi_B(y) = 0$, and $\overline{\chi}_B(y) = 1$. Thus,
$\chi_B(x-y) = 0 \geq \{\chi_B(x) \wedge \chi_B(y)\}$
$\overline{\chi}_B(x-y) = 1 \leq \{\overline{\chi}_B(x) \vee \overline{\chi}_B(y)\}$.
iii. If $x \notin B$, $y \in B$, then $\chi_B(x) = 0$, $\chi^{-}{}_B(x) = 1$ and $\chi_B(y) = 1$ and $\chi^{-}{}_B(y) = 0$. Thus,
$\chi_B(x-y) = 0 \geq \{\chi_B(x) \wedge \chi_B(y)\}$.
$\overline{\chi}_B(x-y) = 1 \leq \{\overline{\chi}_B(x) \vee \overline{\chi}_B(y)\}$.
iv. If $x \notin B$, $y \notin B$, then $\chi_B(x) = 0$, $\overline{\chi}_B(x) = 1$, $\chi_B(y) = 0$ and $\overline{\chi}_B(y) = 1$. Thus,
$\chi_B(x-y) \geq 0 = \{\chi_B(x) \wedge \chi_B(y)\}$.
$\overline{\chi}_B(x-y) \leq 1 = \{\overline{\chi}_B(x) \vee \overline{\chi}_B((y)\}$.

Thus (i) of Definition 3.1 holds good.

Let $x, y \in B$. From the hypothesis, $y + x-y \in B$.
i. If $x, y \in B$, then $\chi_B(x) = 1$, $\overline{\chi}_B(x) = 0$, $\chi_B(y) = 1$ and $\overline{\chi}_B(y) = 0$. In this case,
$\chi_B(y + x-y) = 1 \geq \chi_B(x)$.
$\overline{\chi}_B(y + x-y) = 0 \leq \overline{\chi}_B(x)$.
ii. If $x \in B$, $y \notin B$, th $\overline{\chi}_B$en $\chi_B(x) = 1$, $\overline{\chi}_B(x) = 0$, $\chi_B(y) = 0$ and $\overline{\chi}_B(y) = 1$. Thus,
$\chi_B(y + x-y) = 0 \geq \chi_B(x)$.
$\overline{\chi}_B(y + x-y) = 1 \leq \overline{\chi}_B(x)$.
iii. If $x \notin B$, $y \in B$, then $\chi_B(x) = 0$, $\overline{\chi}_B(x) = 1$, $\chi_B(y) = 1$ and $\overline{\chi}_B(y) = 0$. Thus,
$\chi_B(y + x-y) = 0 \geq \chi_B(x)$.
$\overline{\chi}_B(y + x-y) = 1 \leq \overline{\chi}_B(x)$.
iii. If $x \notin B$, $y \notin B$, then $\chi_B(x) = 0$, $\overline{\chi}_B(x) = 1$, $\chi_B(y) = 0$ and $\overline{\chi}_B(y) = 1$.Thus,
$\chi_B(y + x-y) \geq 0 = \chi_B(x)$.
$\overline{\chi}_B(y + x-y) \leq 1 = \overline{\chi}_B(x)$.

Thus (ii) of Definition 3.1 holds good.

Let $x, y, z \in B$ and $\alpha, \beta \in \Gamma$. From the hypothesis, $x\ \alpha\ y\ \beta\ z$, $x\ \alpha\ (y + z) - x\ \alpha\ z \in B$.
i. If $x, z \in B$, then $\chi_B(x) = 1$, $\overline{\chi}_B(x) = 0$, $\chi_B(z) = 1$ and $\overline{\chi}_B(z) = 0$. Thus,
$\chi_B(\mu((x\ \alpha\ y\ \beta\ z) \wedge (x\ \alpha\ (y + z) - x\ \alpha\ z))) = 1 \geq \{\chi_B(x) \wedge \chi_B(z)\}$.
$\overline{\chi}_B(\mu((x\ \alpha\ y\ \beta\ z) \vee (x\ \alpha\ (y + z) - x\ \alpha\ z))) = 0 \leq \{\overline{\chi}_B(x) \vee \overline{\chi}_B(z)\}$.
ii. If $x \in B$, $z \notin B$, then $\chi_B(x) = 1$, $\overline{\chi}_B(x) = 0$, $\chi_B(z) = 0$ and $\overline{\chi}_B(z) = 1$. Thus,
$\chi_B(\mu((x\ \alpha\ y\ \beta\ z) \wedge (x\ \alpha\ (y + z) - x\ \alpha\ z))) = 0 \geq \{\chi_B(x) \wedge \chi_B(z)\}$.
$\overline{\chi}_B(\mu((x\ \alpha\ y\ \beta\ z) \vee (x\ \alpha\ (y + z) - x\ \alpha\ z))) = 1 \leq \{\overline{\chi}_B(x) \vee \overline{\chi}_B(z)\}$.
iii. If $x \notin B$, $z \in B$, then $\chi_B(x) = 0$, $\overline{\chi}_B(x) = 1$, $\chi_B(z) = 1$ and $\overline{\chi}_B(z) = 0$. Thus,
$\chi_B(\mu((x\ \alpha\ y\ \beta\ z) \wedge (x\ \alpha\ (y + z) - x\ \alpha\ z))) = 0 \geq \{\chi_B(x) \wedge \chi_B(z)\}$.
$\overline{\chi}_B(\mu((x\ \alpha\ y\ \beta\ z) \vee (x\ \alpha\ (y + z) - x\ \alpha\ z))) = 1 \leq \{\overline{\chi}_B(x) \vee \overline{\chi}_B(z)\}$.
iv. If $x \notin B$, $z \notin B$, then $\chi_B(x) = 0$, $\overline{\chi}_B(x) = 1$, $\chi_B(z) = 0$ and $\overline{\chi}_B(z) = 1$. Thus,
$\chi_B(\mu((x\ \alpha\ y\ \beta\ z) \wedge (x\ \alpha\ (y + z) - x\ \alpha\ z))) \geq 0 = \{\chi_B(x) \wedge \chi_B(z)\}$.
$\overline{\chi}_B(\mu((x\ \alpha\ y\ \beta\ z) \vee (x\ \alpha\ (y + z) - x\ \alpha\ z))) \leq 1 = \{\overline{\chi}_B(x) \vee \overline{\chi}_B(z)\}$.

Thus (iii) of Definition 3.1 holds good.

Conversely, suppose that IFS $\overline{B} = <\chi_B, \overline{\chi}_B>$ is an intuitionistic fuzzy ideal of M. Then by Lemma 3.3, χ_B is two-valued, Hence B is a bi-ideal of M.

This completes the proof.

Theorem 3.5.: If $\{A_i\}_{i\in\Lambda}$ is a family of intuitionistic fuzzy bi-ideals of M then $\cap A_i$ is an intuitionistic fuzzy bi-ideals of M, where $\cap A_i = \{\Lambda\mu_{Ai}, V\nu_{Ai}\}$,

$\Lambda\mu_{Ai}(x) = \inf\{\mu_{Ai}(x) | i \in \Lambda, x \in M\}$ and $V\nu_{Ai}(x) = \sup\{\nu_{Ai}(x) \mid i \in V, x \in M\}$.

Proof: Let $x, y \in M$. Then we have,
$\Lambda\mu_{Ai}(x - y) = \inf\{\{\mu_{Ai}(x) \wedge \mu_{Ai}(y)\}|i \in \Lambda, x, y \in M\}$
$= \{\{\inf(\mu_{Ai}(x)) \wedge \inf(\mu_{Ai}(y))\}|i \in \Lambda, x, y \in M\}$
$= \{\{\inf(\mu_{Ai}(x)|i \in \Lambda, x \in M)\} \wedge \{\inf(\mu_{Ai}(y)|i \in \Lambda, y \in M)\}\}$
$= \{\Lambda\mu_{Ai}(x) \wedge \Lambda\mu_{Ai}(y)\}$.

Let $x, y \in M$. Then we have,
$\Lambda\mu_{Ai}(y + x-y) = \inf\{\mu_{Ai}(x)|i \in \Lambda, x, y \in M\}$
$= \Lambda\mu_{Ai}(x)$.

Let $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.
$\Lambda\mu_{Ai}((x\ \alpha\ y\ \beta\ z) \wedge (x\ \alpha\ (y + z) - x\ \alpha\ z)) = \inf\{\{\mu_{Ai}(x) \wedge \mu_{Ai}(z)\}|i \in \Lambda, x, z \in M\}$
$= \{\{\inf(\mu_{Ai}(x)) \wedge \inf(\mu_{Ai}(z))\}|i \in \Lambda, x, z \in M\}$
$= \{\{\inf(\mu_{Ai}(x)|i \in \Lambda, x \in M)\} \wedge \{\inf(\mu_{Ai}|i \in \Lambda, z \in M)\}\}$
$= \{\Lambda\mu_{Ai}(x) \wedge \Lambda\mu_{Ai}(z)\}$.

Let $x, y \in M$. Then we have,
$V\nu_{Ai}(x - y) = \sup\{\{\nu_{Ai}(x) \vee \nu_{Ai}(y)\}|i \in V, x, y \in M\}$
$= \{\{\sup(\nu_{Ai}(x)) \vee \sup(\nu_{Ai}(y))\}|i \in V, x, y \in M\}$
$= \{\{\sup(\nu_{Ai}(x)|i \in V, x \in M)\} \vee \{\sup(\nu_{Ai}(y)|i \in V, y \in M)\}\}$
$= \{V\nu_{Ai}(x) \vee V\nu_{Ai}(y)\}$.

Let $x, y \in M$. Then we have,
$V\nu_{Ai}(y + x-y) = \sup\{\nu_{Ai}(x)|i \in V, x, y \in M\}$
$= V\nu_{Ai}(x)$.

Let x, y, z ∈ M and α, β ∈ Γ.

$V\nu_{Ai}((x\alpha y\beta z) \vee (x\alpha(y+z) - x\alpha z)) = \sup\{\{\nu_{Ai}(x) \vee \nu_{Ai}(z)\} | i \in V, x, z \in M\}$

$= \{\{\sup(\nu_{Ai}(x)) \vee \sup(\nu_{Ai}(z))\} | i \in V, x, z \in M\}$

$= \{\{\sup(\nu_{Ai}(x) | i \in V, x \in M)\} \vee \{\sup(\nu_{Ai}(z) | i \in \Lambda, z \in M)\}\}$

$= \{V\nu_{Ai}(x) \vee V\nu_{Ai}(z)\}$.

Hence, $\cap A_i = \{\Lambda\mu_{Ai}, V\nu_{Ai}\}$ is an intuitionistic fuzzy bi-ideal of M.

Theorem 3.6.: If A is an intuitionistic fuzzy bi-ideal of M then A′ is also an intuitionistic fuzzy bi-ideal of M.

Proof: Let x, y ∈ M. We have,

$\mu'_A(x-y) = 1 - \mu_A(x-y)$

$= 1 - \{\mu_A(x) \wedge \mu_A(y)\}$,

$\nu'_A(x-y) = 1 - \nu_A(x-y)$

$= 1 - \{\nu_A(x) \vee \nu_A(y)\}$.

Let x, y ∈ M. We have,

$\mu'_A(y + x - y) = 1 - \mu_A(y + x - y)$

$= 1 - \mu_A(x)$

$= \mu'_A(x)$,

$\nu'_A(y + x - y) = 1 - \nu_A(y + x - y)$

$= 1 - \nu_A(x)$

$= \nu'_A\, \nu(x)$.

Let x, y, z ∈ M and α, β ∈ Γ. We have

$\mu'_A((x\alpha y\beta z) \wedge (x\alpha(y+z) - x\alpha z)) = 1 - \mu_A((x\alpha y\beta z) \wedge (x\alpha(y+z) - x\alpha z))$

$= 1 - \{\mu_A(x) \wedge \mu_A(z)\}$

$= \{1 - \mu_A(x) \wedge 1 - \mu_A(z)\}$

$= \{\mu'_A(x) \wedge \mu'_A(z)\}$,

$\nu'_A((x\alpha y\beta z) \vee (x\alpha(y+z) - x\alpha z)) = 1 - \nu_A((x\alpha y\beta z) \vee (x\alpha(y+z) - x\alpha z))$

$= 1 - \{\nu_A(x) \vee \nu_A(z)\}$

$= \{1 - \nu_A(x) \vee 1 - \nu_A(z)\}$

$= \{\nu'_A\, \nu(x) \vee \nu'_A\, \nu(z)\}$.

Therefore, A′ is also an intuitionistic fuzzy bi-ideal of M.

Theorem 3.7: An IFS A of M is an intuitionistic fuzzy bi-ideal of M if and only if the level sets

$U(\mu_A; t) = \{x \in M | \mu(x) \geq t\}$ and

$L(\nu_A; t) = \{x \in M: \nu_A(x) \leq t\}$ are a bi-ideal of M when it is non-empty.

Proof: Let A be an intuitionistic fuzzy bi-ideal of M.

Then $\mu_A(x-y) \geq \{\mu_A(x) \wedge \mu_A(y)\}$.

$x, y \in U(\mu_A; t) \Rightarrow \mu_A(x) \geq t, \mu_A(y) \geq t$

$\mu_A(x-y) \geq \{\mu_A(x) \wedge \mu_A(y)\} \geq t$

$\mu_A(x-y) \geq t$

$\Rightarrow x-y \in U(\mu_A; t)$.

$\mu_A(x-y) \geq \{\mu_A(x) \wedge \mu_A(y)\}$.

$x, y \in L(\nu_A; t) \Rightarrow \nu_A(x) \leq t, \nu_A(y) \leq t$

$\nu_A(x - y) \leq \{\nu_A(x) \vee \nu_A(y)\} \leq t$

$\nu_A(x - y) \leq t$

$\Rightarrow x - y \in L(\nu_A; t)$.

Let $\mu_A(y + x-y) \geq \mu_A(x)$.

$x, y \in U(\mu_A; t) \Rightarrow \mu_A(x) \geq t, \mu_A(y) \geq t$

$\mu_A(y + x-y) \geq \mu_A(x) \geq t$

$\mu_A(y + x-y) \geq t$

$\Rightarrow y + x-y \in U(\mu_A; t)$.

Let $\nu_A(y + x-y) \leq \nu_A(x)$.

$x, y \in L(\nu_A; t) \Rightarrow \nu_A(x) \leq t, \nu_A(y) \leq t$

$\nu_A(y + x-y) \leq \nu_A(x) \leq t$

$\nu_A(y + x-y) \leq t$

$\Rightarrow y + x-y \in L(\nu_A; t)$.

Also, let,

$\mu_A((x\alpha y\beta z) \vee (x\alpha(y+z) - x\alpha z)) \geq \{\mu_A(x) \wedge \mu_A(z)\}$.

$x, y, z \in U(\mu_A; t), \alpha, \beta \in \Gamma \Rightarrow \mu_A(x) \geq t, \mu_A(y) \geq t, \mu_A(z) \geq t$

$\mu_A((x\alpha y\beta z) \wedge (x\alpha(y+z) - x\alpha z)) \geq \{\mu_A(x) \wedge \mu_A(z)\} \geq t$

$\mu_A((x\alpha y\beta z) \wedge (x\alpha(y+z) - x\alpha z)) \geq t$

$\Rightarrow (x\alpha y\beta z), (x\alpha(y+z) - x\alpha z) \in U(\mu_A; t)$.

Thus, $U(\mu_A; t)$ is a bi-ideal of M.

$\nu_A((x\alpha y\beta z) \vee (x\alpha(y+z) - x\alpha z)) \leq \{\nu_A(x) \vee \nu_A(z)\}$.

$x, y, z \in L(\nu_A; t), \alpha, \beta \in \Gamma \Rightarrow \nu_A(x) \leq t, \nu_A(y) \leq t, \nu_A(z) \leq t$

$\nu_A((x\alpha y\beta z) \vee (x\alpha(y+z) - x\alpha z)) \leq \{\nu_A(x) \vee \nu_A(z)\} \leq t$

$\nu_A((x\alpha y\beta z) \vee (x\alpha(y+z) - x\alpha z)) \leq t$

$\Rightarrow (x\alpha y\beta z), (x\alpha(y+z) - x\alpha z) \in L(\nu_A; t)$.

Thus, $L(\nu_A; t)$ is a bi-ideal of M.

Conversely, if $U(\mu_A; t)$ is a bi-ideal of M let $t = \{\mu_A(x) \wedge \mu_A(y)\}$. Then

$x, y \in U(\mu_A; t), \Rightarrow x - y \in U(\mu_A; t)$

$\Rightarrow \mu_A(x-y) \geq t$

$\Rightarrow \mu_A(x-y) \geq \{\mu_A(x) \wedge \mu_A(y)\}$.

Also, $x, y \in U(\mu_A; t), \Rightarrow y + x - y \in U(\mu_A; t)$

$\Rightarrow \mu_A(y + x-y) \geq \mu_A(x)$.

If $L(\nu_A; t)$ is a bi-ideal of M let $t = \{\nu_A(x) \vee \nu_A(y)\}$. Then

$x, y \in L(\nu_A; t), \Rightarrow x - y \in L(\nu_A; t)$

$\Rightarrow \nu_A(x - y) \leq t$

$\Rightarrow \nu_A(x - y) \leq \{\nu_A(x) \vee \nu_A(y)\}$.

Also, $x, y \in L(\nu_A; t), \Rightarrow y + x - y \in L(\nu_A; t)$

$\Rightarrow \nu_A(y + x - y) \leq \nu_A(x)$.

Next, define $t = \{\mu_A(x) \wedge \mu_A(z)\}$. Then

$x, y, z \in U(\mu; t), \alpha, \beta \in \Gamma \Rightarrow (x\alpha y\beta z), (x\alpha(y+z) - x\alpha z) \in U(\mu_A; t)$

$\Rightarrow \mu_A((x\alpha y\beta z) \wedge (x\alpha(y+z) - x\alpha z)) \geq t$

$\Rightarrow \mu_A((x\alpha y\beta z) \wedge (x\alpha(y+z) - x\alpha z)) \geq \{\mu(x) \wedge \mu(z)\}$.

Next, define $t = \{\nu_A(x) \vee \nu_A(z)\}$. Then

$x, y, z \in U(\mu; t), \alpha, \beta \in \Gamma \Rightarrow (x\alpha y\beta z), (x\alpha(y+z) - x\alpha z) \in L(\nu_A; t)$

$\Rightarrow \nu_A((x\alpha y\beta z) \vee (x\alpha(y+z) - x\alpha z)) \leq t$

$\Rightarrow \nu_A((x\alpha y\beta z) \vee (x\alpha(y+z) - x\alpha z)) \leq \{\nu_A(x) \vee \nu_A(z)\}$.

Consequently, A is an intuitionistic fuzzy bi-ideal of M.

Theorem 3.8: Let A be an intuitionistic fuzzy bi-ideal of M. If M is completely regular, then $\mu_A(a) = \mu_A(a\alpha a)$, $\nu_A(a) = \nu_A(a\alpha a)$ for all a ∈ M and α ∈ Γ.

Proof: Straight forward.

Let f be mappings from a set X to Y, and A be IFS on Y. Then the preimage of μ under f, denoted by $f^{-1}(A)$, is defined by:

$f^{-1}(\mu_A(x)) = \mu_A(f(x))$, $f^{-1}(\nu_A(x)) = \nu_A(f(x))$ for all x ∈ X.

Theorem 3.9: Let the pair of mappings f: M → N be a homomorphism of Γ-near-rings.

If μ is an intuitionistic fuzzy bi-ideal of N, then the preimage $f^{-1}(A)$ of A under f is an intuitionistic fuzzy bi-ideal of M.

Proof: Let x, y ∈ M. Then we have

$f^{-1}(\mu_A)(x-y) = \mu_A(f(x-y))$

$= \mu_A(f(x) - f(y))$

$\geq \{\mu_A(f(x)) \wedge \mu_A(f(y))\}$
$= \{f^{-1}(\mu_A(x)) \wedge f^{-1}(\mu_A(y))\}$.

$f^{-1}(\nu_A)(x-y) = \nu_A(f(x-y))$
$\nu = \nu_A(f(x)-f(y))$
$\leq \{\nu_A(f(x)) \vee \nu_A(f(y))\}$
$= \{f^{-1}(\nu_A(x)) \vee f^{-1}(\nu_A(y))\}$.

Let x, y ∈ M. Then we have
$f^{-1}(\mu_A)(y + x-y) = \mu_A(f(y + x-y))$
$\geq \mu_A(f(x))$
$= f^{-1}(\mu_A(x))$.

$f^{-1}(\nu_A)(y + x-y) = \nu_A(f(y + x-y))$
$\leq \nu_A(f(x))$
$= f^{-1}(\nu_A(x))$.

Let x, y, z ∈ M and α, β ∈ Γ. Then:
$f^{-1}(\mu_A)((x \alpha y \beta z) \wedge (x \alpha (y + z) - x \alpha z)) = \mu_A(f((x \alpha y \beta z) \wedge (x \alpha (y + z) - x \alpha z)))$
$= \mu_A((f(x \alpha y \beta z)) \wedge (f(x \alpha (y + z) - x \alpha z)))$
$\geq \mu_A(f(x)) \wedge \mu_A(f(z))\}$
$= \{f^{-1}(\mu_A(x)) \wedge f^{-1}(\mu_A(z))\}$.
Therefore, $f^{-1}(\mu_A)$ is an intuitionistic fuzzy bi-ideal of M.

$f^{-1}(\nu_A)((x \alpha y \beta z) \vee (x \alpha (y + z) - x \alpha z)) = \nu_A(f((x \alpha y \beta z) \vee (x \alpha (y + z) - x \alpha z)))$
$= \nu_A((f(x \alpha y \beta z)) \vee (f(x \alpha (y + z) - x \alpha z)))$
$\leq \{\nu_A(f(x)) \vee \nu_A(f(z))\}$
$= \{f^{-1}(\nu_A(x)) \vee f^{-1}(\nu_A(z))\}$.

Therefore, $f^{-1}(\nu_A)$ is an intuitionistic fuzzy bi-ideal of M.

Therefore, $f^{-1}(A)$ is an intuitionistic fuzzy bi-ideal of M.

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