

Teaching Vectors to Junior Secondary School Learners: A Comparison of Two Approaches

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Abstract

The purpose of this study was to investigate whether Form 2 learners taught using the position and movement approach (Experimental group) performed better in a test on Vectors than their counterparts taught using the non-geometric or matrix approach (Control group). A comparison of the two approaches in relation to the Gestalt and the Behaviourist learning theories was also discussed. The mixed research approach (QUANT- Qual) was used. For the QUANT part, the quasi-experimental approach involving the pre-test – post-test control group design with 50 randomly chosen learners (25 in each group) was used. Results showed that, generally, the Experimental group performed better than the Control group, although the findings could be taken with caution due to the small sample sizes and the lack of a true experiment. For the Qual approach, four learners were purposively sampled. The learners were asked to talk about their experiences and performances during learning. An attempt to relate these experiences with the Gestalt and Behaviourist learning theories was also made. It could be concluded that the position and movement approach fitted quite well with the Gestalt Theory, whereas the matrix approach fitted with Behaviourism. These results and findings could help to inform the theory and practice of teaching Vectors in particular and other Math topics in general. There is also a need for further research to see if consistencies in these views could be established.

Keywords: vectors, teaching approach, position and movement approach, matrix approach, gestalt theory, behaviorist theory, form two

Introduction

A lot of research on factors affecting mathematics achievement at secondary school or similar levels has been undertaken the world over. Among the significant factors are the issues of teaching and learning resources, students' attitudes, cognitive ability and anxiety, as well as teaching approaches and methods (Amadalo et al., 2011, Mawarire & Chirume, 2020, Mbugua et al., 2012). Learner-centered approaches in which the student actively interacts with the teacher and with other students in the learning processes may contribute significantly to their achievement in mathematics. However, the question of which approach to choose and how to use it is often problematic to most novice teachers.

Review of the Related Literature

This section covers the aims and objectives of teaching vectors (Zimbabwean context), general methods and approaches to teaching mathematics and specific approaches to teaching vectors and students' challenges.

According to the Ministry of Primary and Secondary Education (2015, pp. 1-2) aims of teaching mathematics include "... to enable learners to develop an understanding of mathematical concepts and processes in a way that encourages confidence, enjoyment and interest" and also "to apply mathematics in other learning areas and in life." The objectives of teaching vectors at the Form 2 level are such that learners will be required to, among other things, define a vector, interpret vector notation, identify various types of vectors, represent translation vectors in column form, draw translation vectors on a Cartesian plane and solve problems using

the concept of vectors (p. 35). At the Form 3 level, the objectives will include multiplying a vector by a scalar quantity and finding the magnitude of a vector.

The syllabus aims and objectives are linked to national aims and aspirations in the sense that, for example, learners can be taught theoretically about adding and subtracting vectors and calculating their magnitudes in class but should also be asked to apply such skills outside in the field when calculating lengths of buildings, distances traveled and heights of trees (in short: solving real-world problems) and so on. Thus, the aims of teaching vectors, the educational aims and national goals, in general, are linked in that they all focus on skills and concepts acquisition and development which are necessary for future studies, for linking with other subjects, for productivity, for self-reliance and for economic and national development. For example, learners will be assessed on carrying out geometrical constructions and manipulations accurately and conducting research projects, including those related to enterprise (Ministry of Primary and Secondary Education, 2015, p. 70). Thus, education must be related to productivity and not only mental growth (King, 2011; Little, 1980; Ozturk, 2001)

General Methods and Approaches of Teaching Mathematics

Gill and Kusum (2017) view an approach as a set of ideas or an overall view of looking at things, while a method is a broad approach to doing things (Developing the Lesson Plan, n.d.). For example, some methods of teaching could be the lecture method or the guided discovery method. As regards the teaching of vectors at the Form 2 level, it is assumed that no one specific

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approach has been recommended. Considering that most Form 2 learners (about 14-16 years old) are still adolescents moving from the Piagetian concrete operational stage to the formal or abstract operational one, discovery and child-centered approaches are most suitable. Various methods of teaching mathematics topics (including vectors) have been proposed in the literature. For example, the Ministry of Primary and Secondary Education (2015) lists Guided discovery, Discussion, Interactive e-learning, Exposition, Demonstration and illustration, Problem-solving, Individualisation, Simulation, Visual tactile, Educational tours and Expert guest presentation as some of the methods of teaching mathematics.

Specific Approaches of Teaching Vectors and Students' Challenges

Mai et al. (2017) have come up with geometric and symbolic approaches to the definition of a vector. For example, they stated that based on their secondary school conceptions, some freshmen students defined a vector as "... a quantity with direction and magnitude (geometric) while others defined it as "... an n -tuple (of numbers) such as "... a matrix with n rows and just one column (symbolic) (p. 2190). It may be assumed that a learner may define a mathematical concept in a certain way and carry over such definition(s) in future or later studies. If a wrong definition is not corrected early, the student's achievement in later studies may also be affected.

Mai et al. (2017) have also analyzed students' vector conceptions as they transit from school to university. Students' conceptions were varied but could be classified into four major approaches: symbolic, axiomatic, geometric and 'other.' In their study with 103 first-year university math students, they found out that most of them had a 'geometric basis' of the vector concept as they had learned in high school. Also, the grid method approach (Nur et al., 2017) and abstract approach (e.g., vector as n -tuple and pseudo-abstract (vector as the arrow on a grid with no axes) (Kwon, 2011) have been proposed.

Some secondary school students in Indonesia had problems with the vector concept and also in the representation of vectors in Physics (Jewaru et al., 2021). It was not very clear why, but this could be due to the approaches or contexts in which they had been introduced to the topic.

Even some first-year medical students in Saudi Arabia showed weak overall performance on vectors, with an average test score of 26% and they showed several misconceptions about vectors, even on simple ideas such as addition, subtraction and scalar multiplication (Bani-Salameh et al., 2020). The authors suggested recommendations that point to teaching using graphical representation or a different approach to teaching.

Nur et al. (2017) also established that students' difficulties with vectors lie in their 'vector imaginations' and conceptualizations. They further found out that the grid method approach increased awareness and understanding of the importance of vector concepts.

However, in this study, sufficient and relevant research articles in the literature on the comparison of two different approaches when teaching vectors at the secondary school level, particularly in Zimbabwe, could not be found.

Theoretical Frameworks

Many teaching and learning theories can be used in the teaching and learning of mathematics topics, such as vectors in secondary schools. Some of them are behaviorist-learning theories, cognitive learning theories, constructivism learning theories, social critical learning theories and Gestalt learning theories, among others. In this study, Gestalt and behaviorist learning theories were chosen.

Gestalt Theories of Learning

Gestalt means 'unified whole' in German. According to Interaction Design Foundation (n.d.) Gestalt principles are "... laws of human perception that describe how human beings group similar elements, recognize patterns and simplify complex images when they perceive

objects." It is based on the 'Aha' insight and discovery learning. For instance, learners can have the 'Aha' experience through the use of various learning media and diagrams, which lead to a firm concept of a vector as a directed line segment with a starting point (tail) and endpoint (head). The Gestaltists postulated that the whole is greater than the sum of its parts. So, it is presumed that learners can perform better on the topic of vectors by learning about the whole (vector) by studying its parts (coordinates of its tail, coordinates of its head, direction, magnitude, etc.) and then putting together the parts to come up with the whole.

Behaviorist Theories of Learning

On the other hand, behaviorist theory says learning takes place through interaction with the environment and heredity and thought processes that occur in mind do not have much influence (Ng'andu et al., 2013; Teaching & Education, 2020). Some stimulus is given (e.g., by the teacher) and results in a response (e.g., by the learner giving a correct or wrong answer). The response is rewarded so that it can be repeated, perfected, ignored, or punished so that it can be stopped. (ZOU Module BEDY207, n.d.). So, when teaching vectors, the teacher can encourage and motivate learners to perform better through positive reinforcements such as giving comments like 'good, keep it up, you are a star' and so on. However, there are some thought processes that can occur in the mind and behaviorism does not pay much attention to them. Apart from the environment, heredity is also believed to have some influence on student learning (Bueno, 2019).

Background to the Problem

Throughout his teaching experiences, the researcher has noticed that many mathematics teachers usually fail to introduce or present the subject matter using an appropriate (or the best) approach so that learners fully master the related concepts and skills right up to the end of the teaching unit or topic. The teachers might have knowledge of the teaching method and/or content at hand, but it has also been established that the use of the teacher's instructional plan (Amadalo et al., 2011) and the way a topic is introduced (Waxman, 1987) have a bearing on learners' outcomes. Also, according to Developing the Lesson Plan (n.d.), a good lesson introduction determines the level of learners' motivation, recapitulation, recall and feedback, exposes benefits to the students and neatly leads to the development (content) and conclusion of the lesson. In Zimbabwe, for instance, the topic 'Vectors' is first introduced at the junior secondary level (Form 2) and covers the following subtopics: Definition of vector, Vector notation, Types of Vectors (Translation vectors, Negative vectors, Equal vectors, Parallel vectors), and Addition and Subtraction of vectors (Ministry of Primary and Secondary Education, 2015). The scalar multiplication and magnitude of a vector have since been moved to the Form 3 level. The methods and/or approaches to use during a particular lesson or series of lessons are left for the teacher to decide. Nevertheless, when introducing and teaching the topic 'Vectors' at the Form 2 level, some teachers often have the problem of determining and using the 'best' approach out of several ones, such as position and movement, matrix, translation, physical and the free vector approaches.

Purpose of the Study

The purpose of this study is to investigate whether learners taught using the position and movement approach perform better in a test on Vectors than their counterparts taught using the non-geometric or matrix approach. A comparison of the two approaches in relation to the Gestalt and the Behaviourist learning theories is also given.

Research Questions

1. Is the performance of Form 2 learners who are taught using the position and movement approach (A) significantly different from the performance of their counterparts who are taught using the matrix approach (B)?

2. Is approach B (or A) more suitable and effective than approach A (or B) at that level? In other words, 'Is the situation reflective of the population as a whole, or is it generalizable?'
3. How
4. the two approaches are compared in relation to the Gestalt and the Behaviourist theories of learning?
5. What likely changes should be implemented and what would be their anticipated effects?

Hypotheses

1. Ho: $\bar{x}_1 = \bar{x}_2$ versus H1: $\bar{x}_1 \neq \bar{x}_2$ at 5% significance level
2. Ho: $\bar{\mu}_1 = \bar{\mu}_2$ versus H1: $\bar{\mu}_1 \neq \bar{\mu}_2$ Or H1: $\bar{\mu}_1 > \bar{\mu}_2$ at 5% significance level

Methodology

Paradigm and Design

The 'mixed methods' paradigm was used, which included the quantitative approach predominantly (QUANT), followed to a lesser extent by the qualitative approach (Qual). For the Qual part, four learners were purposively sampled. The learners were asked to talk about their experiences and performances during learning. An attempt to link these experiences with the Gestalt and Behaviourist learning theories was also made. For the QUANT part, the quasi-experimental design was used whereby two groups of Form 2 learners were chosen and randomized. A pre-test was given to all of them. Different treatments were given to Groups A and B (Control and Experimental, respectively). After the treatments, the two groups were post-tested. Thus, the pre -test-post-test control group design was used:

R O1 X1 O2
R O3 O2

Data Collection Procedure

The plan and procedure were as follows:

1. The learners were sampled using simple random sampling and random numbers generated by a scientific calculator.
2. Two groups of 25x2=50 learners – comprising low, medium and high-performing learners, were created. Group A was the control group, while Group B was the experimental one. The students' code names or ID's and their gender were recorded.
3. A ten-item, 20-minute-long pre-test was prepared and pre-tested to all Form Two learners on addition, subtraction and scalar multiplication of vectors.
4. The same pre-test was given to the two groups, marked and the test scores recorded. The code name/ID and gender of each student were also captured.
5. Group A was taught using the Matrix Approach (vector as a column matrix- 'symbolic'), while Group B was taught using the

Position and Movement Approach (vector as directed line segment- 'geometric').

6. A ten-item, 20 minute- long post-test was prepared and post-tested to all Form Two learners (Groups A and B) on addition, subtraction and scalar multiplication of vectors.
7. The post-test was marked, marks recorded, and student's code names/ID and gender were also recorded for each student.
8. For the 'Qual' part, views and opinions solicited from the sampled learners were transcribed on paper and analyzed with regard to Gestalt and behaviorist theories.

Data Presentation and Analysis Procedure

Descriptive statistics (means, standard deviations and correlations) were used to present and analyze data. The *t*-test was also used to analyze data and draw inferences and conclusions pertaining to the hypotheses. Data were also analyzed qualitatively through a critical reflection on the methods used and approaches used and the learners' general participation and performance. A comparison/reflection on the Gestalt and Behaviourist learning theories was also undertaken via an analysis of learners' voices which were transcribed from the interview data.

Results and Discussion

Quantitative Data

Table 1
Means and Standard Deviations of Pre-test and Post-test for Groups A and B

Group	M	SD
Pre-test A	21.60	8.21
Pre-test B	27.50	7.39
Post-test A	18.60	7.34
Post-test B	22.80	11.28

Note. N = 25

Table 1 shows some noticeable differences between the means for the pre and post-tests for the Control (A) and Experimental (B) groups. The means for Group B were bigger than the means for Group A. From the raw scores, the marks for Group A were concentrated between 38% and 58%, while those for Group B were concentrated between 20% and 80%. Also, the post-test results were poorer than the pre-test results, with Group B's results being slightly better than Group A's. Whether these differences were significant or not will be determined by the *t*-tests (see Table 2).

Table 2
Pre-test and Post-test Correlations for Groups A and B

Group	Statistic	Pre-test A	Pre-test B	Post-test A	Post-test B
Pre-test A	Pearson correlation	1	-.098	.50*	-.022
	Sig. (2-tailed)		.64	.010	.92
	N	25	25	25	25
Pre-test B	Pearson correlation	-.098	1	-.34	.62**
	Sig. (2-tailed)	.64		.09	.001
	N	25	25	25	25
Post-test A	Pearson correlation	.50*	-.34	1	-.46*
	Sig. (2-tailed)	.01	.09		.022
	N	25	25	25	25
Post-test B	Pearson correlation	-.022	.62**	-.46*	1
	Sig. (2-tailed)	.92	.001	.022	
	N	25	25	25	25

Note. * Correlation is significant at the .05 level (2-tail); ** Correlation is significant at the .01 level (2-tailed).

The results of Table 2 for the correlational analysis were as follows: There was a significant moderate positive correlation between the Pre-test and Post-test scores for Control Group A ($r = .50, \alpha = .01, 2$ -tailed). There was also a significant positive moderate to a high correlation between the Pre-test and the Post-test scores

for Experimental Group B ($r = .62, \alpha = .001, 2$ -tailed). This could also mean that the Post- and Pre-tests were not very different from each other. There was a significant negative moderate to low correlation between the Post-test results for Groups A and B ($r = -.46, \alpha = .022, 2$ -tailed). The other correlations were not significant at the 5% level.

Table 3
T-Tests

Pair	Group	M	SD	SEM	95% CI of difference		t	df	Sig (2-tailed)
					Lower	Upper			
Pair 1	Pre A-Pre B	-5.96	11.574	2.315	-10.737	-1.1825	-2.575	24	.017
Pair 2	Pre A-Post A	3.00	7.78	1.556	-.213	6.21288	1.927	24	.066
Pair 3	Post A-Post B	-4.2	16.026	3.205	-10.815	2.415	-1.31	24	.202
Pair 4	Pre B-Post B	4.76	8.866	1.773	1.100	8.4197	2.864	24	.013

Note. Pre A means the Pre-test for Group A

Table 3, *t*-tests results show that there were significant differences between the performance of the Control and Experimental Groups, of which the Experimental Group performed better ($t = -2.575, \alpha = .017$). Not to be expected was the scenario that the pre-test results for the Experimental Group were significantly better than the post-test results for the same group ($t = 2.864, \alpha = .013$). Maybe the post-test seemed harder to the pupils than the pre-test. For the other pairs (Pre-test A vs. Post-test A and Post-test A vs. Post-test B), the *t*-test results were not significant at the 5% level. Thus, the hypothesis $H_0: \bar{x}_1 = \bar{x}_2$ was rejected in favor of $H_1: \bar{x}_1 \neq \bar{x}_2$ at a 5% significance level. However, because of the small sample sizes and not being a truly experimental design, generalization to the larger population could be made with caution (it would be unwise to confidently accept $H_1: \bar{\mu}_1 \neq \bar{\mu}_2$ Or $H_1: \bar{\mu}_1 > \bar{\mu}_2$).

Table 4
Gender differences

Group	M	SD	Max	Min	Range
Pre-test A: Male	21.6	5.68	31	10	21
Pre-test A: Female	21.6	11.38	42	9	33
Pre-test B: Male	27	7.81	43	17	26
Pre-test B: Female	28.17	7.20	43	18	25
Post-test A: Male	18.73	5.19	30	8	22
Post-test A: Female	18.4	10.08	32	6	26
Post-test B: Male	22.85	11.86	40	8	32
Post-test B: Female	22.75	11.38	42	7	35

Note. Min. = minimum; Max. = maximum.

It was considered prudent to investigate possible gender differences of which the following emerged: Although Table 4 shows some small real differences in the average performance of male and female pupils, there were no statistically significant differences in the performance of the groups with respect to gender.

Qualitative Data

A brief discussion and reflection on the Gestalt and Behaviourist learning theories were undertaken with regard to the teaching of mathematics in general and the teaching of vectors in particular.

Since Gestalt psychologists "analyze the entire field of the gestalt in terms of relationships rather than distinctions" (von Meier, 1975, para. 10), it could be worthwhile to think of some factors that could have affected pupils' performance. These factors could be separated from their classroom peers and being grouped into 'artificial' groups, not used to writing pre-tests and sometimes learning after normal hours in order to cover up for the planned work for the study. Hence a study of how the 'Gestalt' units combine helps to understand better the differences in the performances before making conclusions. A variety of teaching methods could also have been used contrary to what had been planned before carrying out the research.

Behaviourism has been the dominant theory for teaching math for many years (Montilla, 2019), but according to Ng'andu et al. (2013, p. 13), "... behaviorism cannot stand on its own as a theory of teaching. Hence, it is best used in conjunction with other methods. Moreover, since behaviorism is based on memorization of tasks by the learner, it is not useful in the teaching of complicated subject matter." Thus "The discussion about the learning theories becomes focused on whether it is wisest to provide an efficient learning environment that results primarily in the acquisition of academic knowledge or to take an approach that provides a more in-depth, indirect process encompassing the whole learner. If a basic concept is not fully comprehended, should the student move on?" (Montilla, 2019, p. 3). These were questions or limitations left unanswered in this research.

However, some selected excerpts from the learners' voices which were as follows, could help to compare and reflect on the Gestalt and behaviorist theories with regard to the teaching of vectors:

I have now understood that there is a relationship between vectors and matrices since a vector is a column matrix (Learner 2, Group B). This fits with the Gestalt theory whereby the learner learns by separating things into units, putting together the units and finding their relationships, etc.

I did not have problems with the idea of a scalar since we had covered this in the previous topic of matrices (Learner 1, Group A). This could be classified into Behaviourism, where learning is transferred from one situation or scenario to another (transfer of learning).

I am not good at drawing, but after my teacher's demonstration, I am now able to draw and represent any vector of any direction and size on the Cartesian plane (Learner 3, Group B). Learner 3 may have had the 'aha' experience resulting from the teacher's demonstration and explanation; hence this could be grouped into the Gestalt learning theory.

At first, I failed to apply Pythagoras' theorem to find the magnitude of the vector $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$. My teacher explained and worked on similar problems on the blackboard. When I had finally solved the problem, the teacher thanked me and encouraged me to keep it up (Learner 4, Group A). One notes that the statement by Learner 4 fits into the Behaviourist theory where he was intrinsically motivated (positive reinforcement).

I could not fully understand word problems on vectors because the English used was too difficult for me (Learner 2, Group B). According to the Gestalt theory of putting together relationships and connections, this learner failed to experience the 'aha' due to language.

In this study, the common method used was 'the teacher demonstrating and questioning and pupils answering and working out problems.' Pupils were taught in one week. Weekly assignments and exercises were different as the approaches for the two groups were different. So, the differences in pupils' performance in the two groups could be due to the different approaches, among other possible reasons. The textbooks used were the New General Mathematics Book 2 (new edition, 1984, chapter 26 & old edition, 1970, chapter 9) by Channon et al, Essential Mathematics for 'O' level (Bolt, 1984) and A First Course in Modern Mathematics Volume II by Marie Anderson, Published by Heinemann Educational Books (1983).

Conclusion

With reference to the pupils' scores, it could be concluded from this research that the pupils taught using the position and movement approach performed better in a test on vectors than those taught using the matrix approach.

However, there was not enough evidence to conclude that approach B (or A) was more suitable and effective than approach A (or B) at that level. Due to the small sample sizes used, generalizations about to populations could be made with caution.

Some distinctions between the Gestalt and Behaviourist learning theories with regard to the teaching of vectors at the Form 2 level were made. This was done by analyzing the comments made by pupils on their learning experiences. From the comments made by the participants, one could conclude that the position and movement approach fitted quite well with the Gestalt Theory, whereas the matrix approach fitted with Behaviourism, although further research could establish otherwise.

Although it was not initially the objective of the research, this study found that there were no significant gender differences in the performance of both groups. In conclusion, the research questions of this study have been answered and the hypotheses have been satisfactorily tested.

Recommendations

It would appear that the practical (displacement-position and movement) approach should be used more than the theoretical approach since adolescent students seem to like practical and game-like activities more than theoretical ones. It is recommended that textbooks that have a variety of pupil-centered activities and a variety of methods to be used by the teachers, such as Channon et al. (1984), should continue to be used. A variety of teaching and learning media should also be used. Further research on teaching methods and approaches with different math topics and at different learners' levels should be carried out.

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